Two Questions and Three Equations on Distance of Investigation

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The distance of investigation concept is often used to answer two different types of questions:

Q1. How far have we investigated into the reservoir? (At what distance in the reservoir has the pressure been disturbed minimally?)

Q2. What is the distance to the boundary?

Traditionally, these two questions have been answered using the same equation, namely, the radius of investigation equation. This equation is $r_{inv} = \sqrt{\frac{kt}{948\phi\mu c}}$ (t is in hours), and is derived from $t_{De} = 0.25$.

This approach has worked well for many years. However, with the appearance of unconventional reservoir systems, which are dominated by linear flow, other distance of investigation equations are being used, and to further confuse the issue, the distance of investigation equations for constant flow rate and constant pressure conditions are different.

There are essentially three equations for distance of investigation in common use:

Eq.1 Radial flow (both constant rate and constant pressure): $t_{De}=0.25$
Eq.2 Linear Flow – constant pressure: $t_{De}=0.25$
Eq.3 Linear Flow – Constant rate: $t_{De}=0.5$

Our investigation into the two questions, and the three equations, has resulted in some clarifications, and some unexpected results. These are published in (Tabatabaie et al, 2017) and summarized in this article. The main findings of our research are:

1. The two questions represent two different concepts and should NOT be answered using the same equation.
   - Q1 (How far have we investigated) needs its own set of equations.
   - Q2 (Distance to boundary) is properly answered using what have been traditionally called "distance of investigation equations", namely Eq1., Eq.2 and Eq.3.
2. When a pressure disturbance is initiated at the well, there are two issues that are relevant:
   a. When does the effect of the disturbance reach a specified location in the reservoir? This is called the **Time of Arrival**.
   b. When can the effect of a boundary be detected at the flowing well? This is called the **Time of Detection**.
3. At the Time of Arrival, the depletion at the boundary is minimal (<5%)

4. At the Time of Detection, the depletion at the boundary is minimal (<5%) for radial flow, but is substantial (30-40%) for linear flow.

Table 1: Summary of Arrival Times, Detection Times and Depletions

Flow System	Arrival		Detection	
	Time	**Depletion***	**Time**	**Depletion***
Fig 3a: Radial – Constant Pressure (Rad-CP)	t_{De}=0.25	<5%	t_{De}=0.25	<5%
Fig 3b: Radial – Constant Rate (Rad-CR)	t_{De}=0.25	<5%	t_{De}=0.25	<5%
Fig 3c: Linear – Constant Pressure (Lin-CP)	t_{De}=0.1	<5%	t_{De}=0.25	30%
Fig 3d: Linear – Constant Rate (Lin-CR)	t_{De}=0.1	<5%	t_{De}=0.5	40%

*Depletion for constant rate and constant pressure cases are defined as $\frac{p_i-p}{p_i}$ and $\frac{p_i-p}{p_i-p_w}$, respectively

5. Time of Detection should be used for determining distance to a boundary
6. Time of Arrival should be used for determining how far the disturbance has travelled into the reservoir.
7. For radial flow, there is little difference between Time of Arrival and Time of Detection, and that is why historically, a single equation has successfully answered both Q1 and Q2; see Figures 4a and 4b.
8. However for linear flow, the Time of Detection is 2.5-5 times the Time of Arrival!; see Figures 4c and 4d.
9. For radial flow the speed of travel of a disturbance is the same for constant rate and for constant pressure – Figure 1a.
10. However for linear flow, the constant pressure pulse appears to travel a little faster than the constant rate pulse – Figures1b and 1c.
11. The “speed of propagation” of a particular depletion depends not only on the hydraulic diffusivity of the medium, but also on the magnitude of the depletion of interest.
12. In summary, the traditional phrase “distance of investigation” being used to represent two different concepts is confusing. To avoid confusion, the phrase “distance of investigation” should be replaced by two different terms, namely:
    - “**distance of influence**”: this corresponds to the **time of arrival** and represents the distance in the reservoir that has been disturbed.
    - “**distance to boundary**”: this corresponds to “**time of detection**” and gives correctly the distance to a boundary (no-flow).

Time of Arrival: The time at which an ***observer located at a point inside the reservoir*** detects a measurable pressure difference, typically a pressure difference of <5%.

- The way to determine Time of Arrival is to calculate the pressure profile **in the reservoir** and note when the pressure at the point of interest declines visibly. See Figures 1a, 1b and 1c.

Time of Detection: The time at which an ***observer located at the flowing well*** detects a deviation from infinite acting behavior (senses the boundary). For radial flow, at the time of detection, depletion at the boundary is <5%. However, for linear flow, at the time of detection, depletion at the boundary is 30-40 %!!

- The way to determine Time of Detection is to plot the flowing pressure **at the well,** and note when it deviates from infinite-acting behavior, see Figures 2a, 2b, 2c and 2d.

Time of Arrival of pressure disturbance at boundary

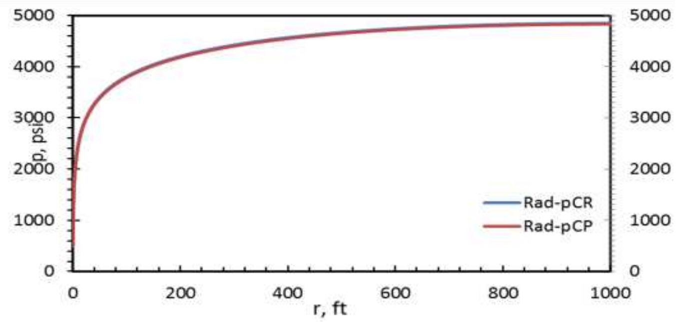

Fig 1a: Radial Flow: Pressure distribution in reservoir for CP and CR

- Both CP and CR curves reach the boundary at $t_{De}=0.25$;
- Depletion at re < 5%
- There is an imperceptible difference between CP and CR for radial flow.

Conclusion: Transients for CR and CP travel at the same speed; $t_{De}=0.25$ (Arrival)

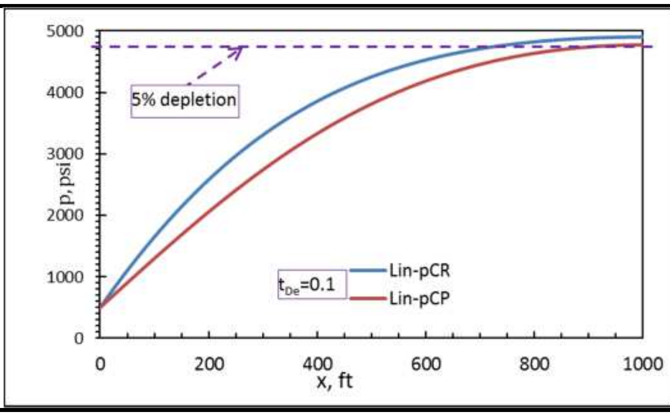

Fig 1b: Linear Flow: Pressure distribution in Reservoir for CP and CR at $t_{De}=0.1$ with flow rate selected so as to give the same p_{wf}

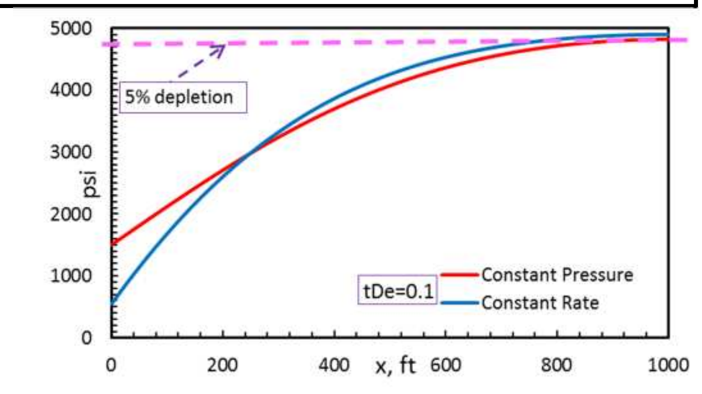

Fig 1c: Linear Flow: Pressure distribution in Reservoir for CP and CR at $t_{De}=0.1$ with flow rate selected so as to give the same cumulative production

- In general, transients for CP travel a little faster than CR, but not significantly so. For practical purposes, $t_{De}=0.1$ is an acceptable time of arrival for both
- At ($t_{De}=0.1$), depletion at x_e is < 5%, for both CP and CR

Conclusion: $t_{De}=0.1$ (Arrival) for both CP and CR; (CP is minimally faster)

Table 2: Time of Arrival of Pressure Disturbance at Boundary	
Flow System	Time of Arrival
Fig1a: Radial – Constant Pressure (Rad-CP)	t_{De}=0.25
Fig1a: Radial – Constant Rate (Rad-CR)	t_{De}=0.25
Fig1b,1c: Linear – Constant Pressure (Lin-CP)	t_{De}=0.10
Fig1b, 1c: Linear – Constant Rate (Lin-CR)	t_{De}=0.10

Discussion of Time of Arrival:
It is evident that the Time of Arrival is dependent on the flow geometry. It is faster for Linear Flow than for Radial Flow. However, if the time of Arrival was expressed in terms of t_{DA} instead of t_{De} (area rather than distance), t_{DA} =0.1 for both Linear Flow and Radial Flow. This is consistent with the fact that Arrival Time is associated with a depletion (<5%), and that the degree of depletion depends on the reservoir area being depleted.

Time of Detection of Boundary at Flowing Well

The **time of detection** is obtained by plotting the pressure at the well as a function of time. There are five common ways of doing that.

a. p_{wD} vs t_{De} on a Cartesian plot
b. log p_{wD} vs log t_{De} (log-log plot) sometime referred to as typecurve plot
c. log Der vs log t_{De} (Derivative plot)
d. **Radial Flow:** p_{wD} vs log t_{De} (semi-log plot), also known as "radial specialized plot"
e. **Linear Flow:** p_{wD} vs Sqrt-t_{De} (square-root plot), also known as "linear specialized plot"

Note that by the very nature of the equations of flow, any distance of investigation is an approximation. The times reported here are necessarily approximate and depend on the scales and method of plotting. The intention here is not to be exact, but to demonstrate that there is a significant difference as the geometry of flow changes. Hence, times of deviation are chosen to be consistent with those commonly accepted in the industry, and are meant to illustrate concepts and relationships rather than be exact answers.

Plots a. (Cartesian) and b. (log-log) are generally not sensitive enough. Plot c. (Derivative) is very sensitive and would work well for well testing (good quality data) but would not be useful for RTA because of the noisy data. Plots c. and d., the specialized plots for each flow regime are the most practical, and will be used here. Thus, **Time of Detection** for **radial** flow is defined as the departure of the data from the semilog straight line (Infinite-Acting-Radial-Flow, IARF), and **Time of Detection** for **Linear** flow is defined as the departure of the data from the square-root straight line (Infinite-Acting-Linear-Flow, IALF).

Time of Detection

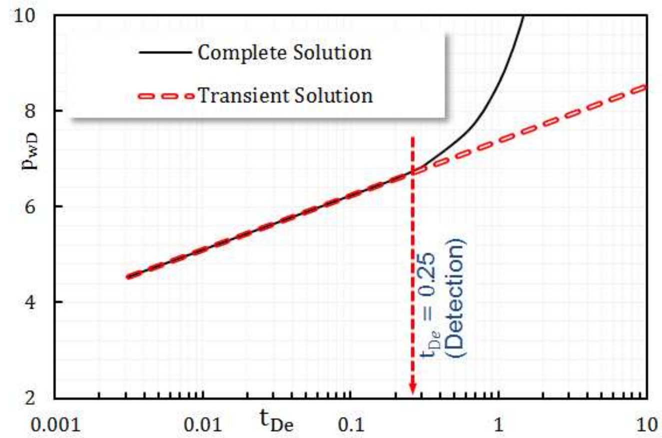

Fig 2a: Radial Flow: Time of Detection –Const Pressure

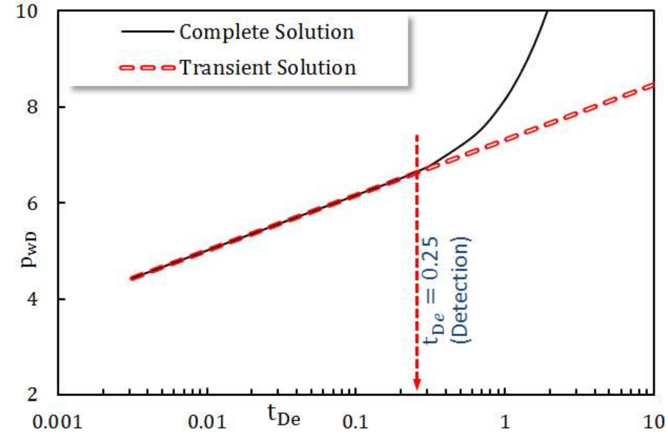

Fig 2b: Radial Flow: Time of Detection –Const Rate

- Deviation from semilog straight line (IARF) is t_{De}=0.25, for both CP and CR

Conclusion: Time of detection for Radial Flow is $t_{De}=0.25$; it is the same as for CP and CR

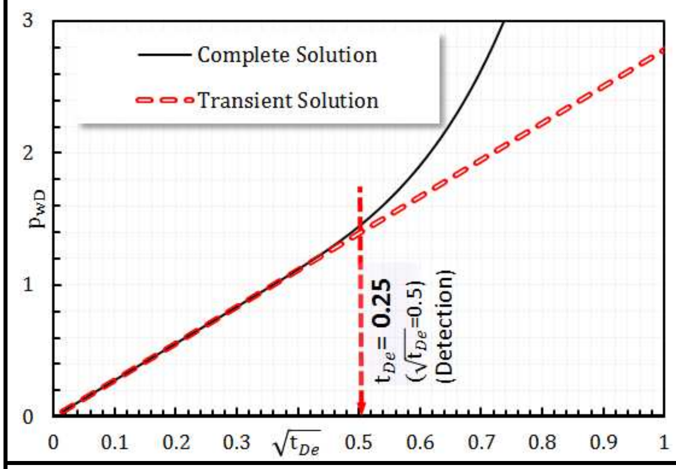

Fig 2c: Linear Flow: Time of Detection –Const Pressure

- Deviation from square-root straight line (IALF) is t_{De}=0.25
- This value is consistent with Wattenbarger's

Conclusion: Time of detection for Linear CP is **t_{De}=0.25**

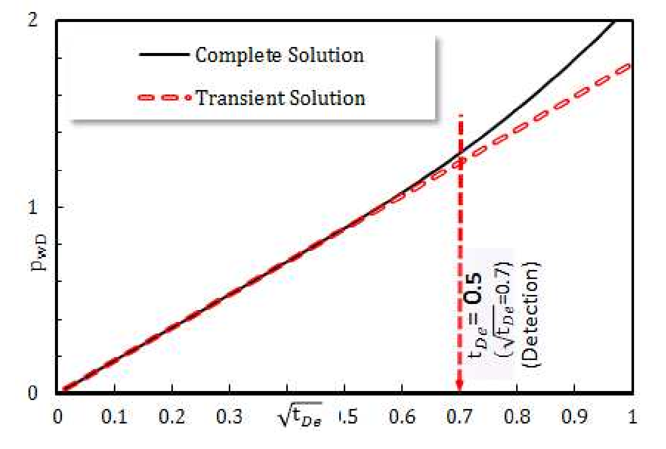

Fig 2d: Linear Flow: Time of Detection–Const Rate

- Deviation from square-root straight line (IALF) is t_{De}=0.5
- This value is consistent with Wattenbarger's

Conclusion: Time of detection for Linear CR is **t_{De}=0.5**

Table 3: Time of Detection of Boundary	
Flow System	Time of Detection
Fig2a: Radial – Constant Pressure (Rad-CP)	t_{De}=0.25
Fig2a: Radial – Constant Rate (Rad-CR)	t_{De}=0.25
Fig2b: Linear – Constant Pressure (Lin-CP)	t_{De}=0.25
Fig2b: Linear – Constant Rate (Lin-CR)	t_{De}=0.5

Discussion of Time of Detection:

The fact that for Linear Flow, the Time of Detection, for constant pressure and constant rate, are significantly different (t_{De}=0.25 and 0.5) is of particular interest, and poses the problem of which equation to use in production situations where rates and pressures vary. This issue appears to be contradictory to the traditionally-accepted concept that the distance of investigation is independent of rate.

The findings of our study are presented below. They are stated with an apparent degree of definitiveness. This is done to aid in clarity of presentation and is not meant to negate the approximate nature of the concept of distance of investigation and the scatter in the data.

1. For radial flow, time of detection is independent of rate.
2. For linear flow, the boundary is detected earlier for Constant Pressure (t_{De}=0.25) than for Constant Rate (t_{De}=0.50). An explanation is that for these two different situations, we are tracking different variables, and depending on what we are tracking (pressure or rate) we will see the effect of the boundary at different times.
3. In the constant rate scenario we are tracking $\frac{dp_w}{dt}$ while in the constant pressure scenario we are tracking $\frac{d\frac{1}{q}}{dt}$, and they are different by a factor of $\frac{\pi}{2} = 1.6$. This factor is consistent with the ratio of t_{De}=0.50 : t_{De}=0.25 (remember these numbers are approximate and are rounded to reflect numbers in common use. If we had used the intersection of derivatives (which seems to be more sensitive in showing the changes but unfortunately is too noisy to be used in practice, t_{De}(CR)=0.32 and t_{De}(CP)=0.2, and the ratio will be 1.6.).

$$\frac{1}{q_D} = \frac{\pi}{2}\sqrt{\pi t_D} \quad \Rightarrow \quad \frac{d\left(\frac{1}{q_D}\right)}{dt_D} = \frac{\pi}{2}\frac{\sqrt{\pi}}{2\sqrt{t_D}}$$

$$p_D = \sqrt{\pi t_D} \quad \Rightarrow \quad \frac{dp_D}{dt_D} = \frac{\sqrt{\pi}}{2\sqrt{t_D}}$$

4. From Fig 3a, it is clear that Time of Detection for constant pressure is earlier than for constant rate. However when these data sets are plotted versus Material

Balance Time (t_{MB}=Cumulative/q), as shown in Figure 3b, these differences disappear (for all practical purposes.(Note, for constant rate, t_{MB}=t)

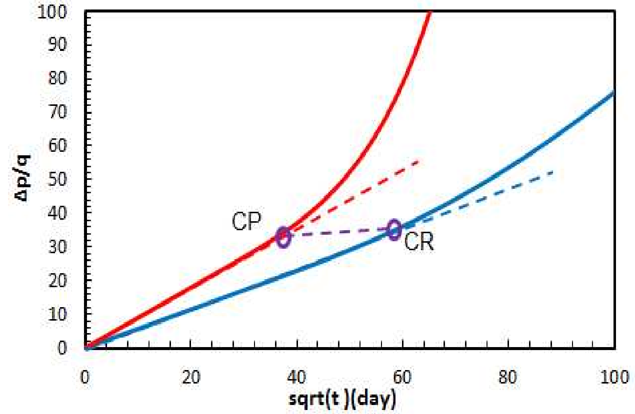

Fig.3a: Sqrt time plot: Red: CP; Blue: CR

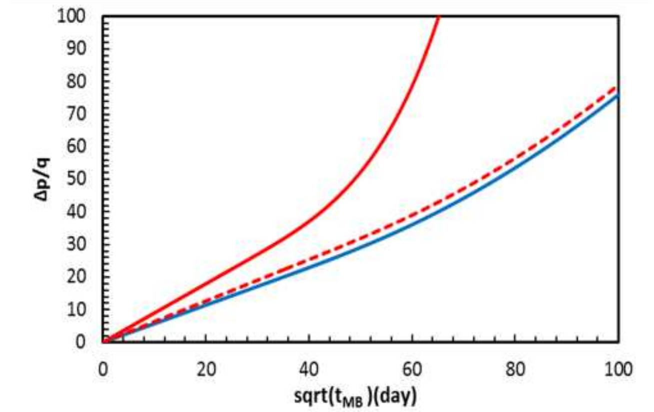

Fig.3b: Sqrt Material Balance Time plot: Dashed Red: CP; Blue: CR (solid red is t, not t_{MB})

5. if Material Balance Time is used in place of time, the equation for Time of Detection for both CP and CR is unified to a single constant value of t_{De}=0.5, for all practical purposes.

Combined Pictorial of Time of Arrival and Time of Detection

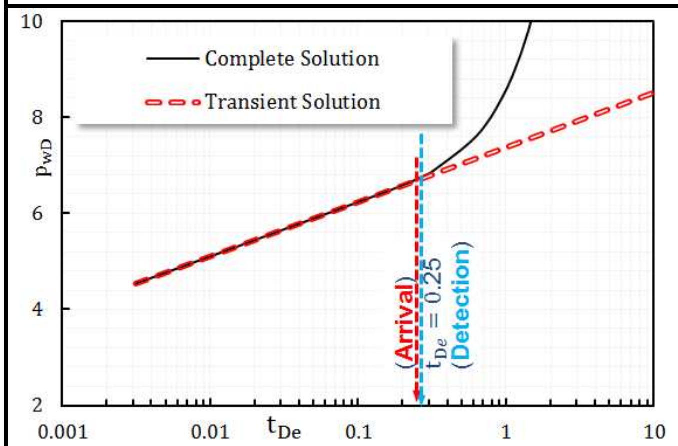

Fig 4a: Radial CP: Arrival and Detection

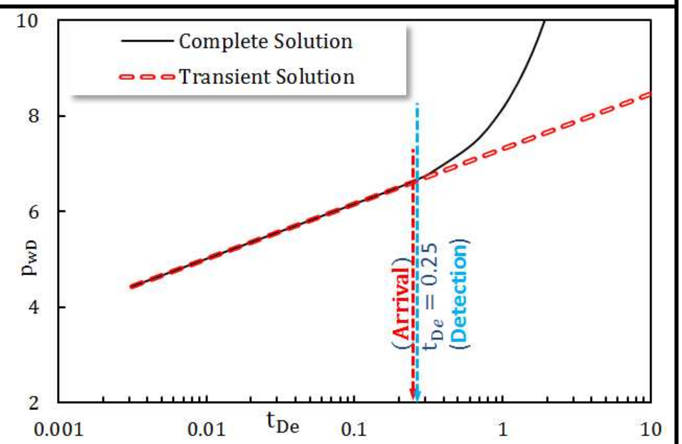

Fig 4b: Radial CR: Arrival and Detection

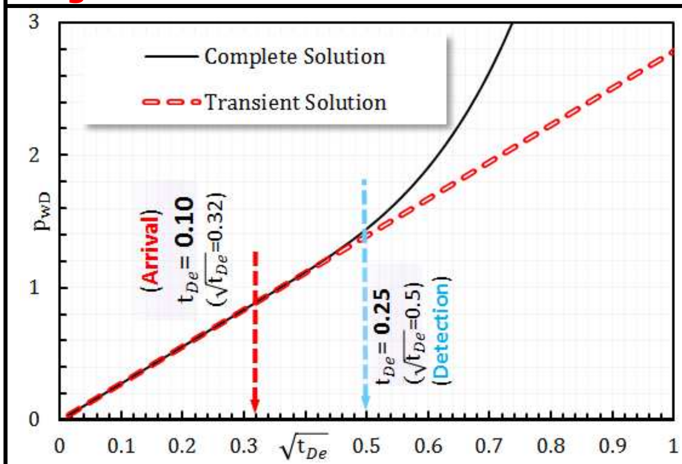

Fig 4c: Linear CP: Arrival and Detection

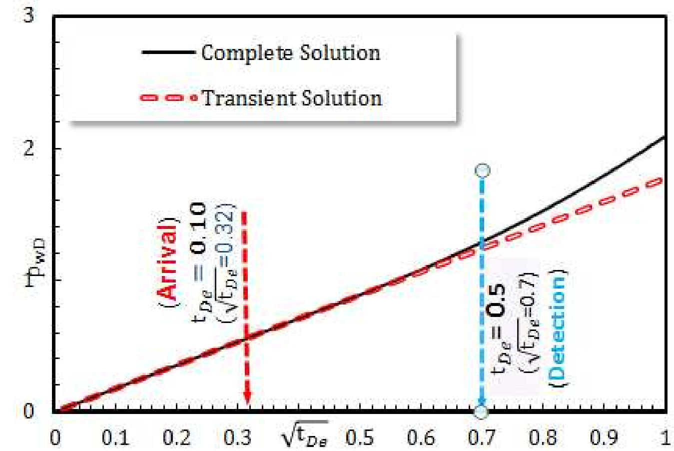

Fig 4d: Linear CR: Arrival and Detection

Table 4: Combined Tabulation of Arrival Times and Detection Times			
Flow System		Arrival	Detection
Fig 4a: (Rad-CP)	Radial – Constant Pressure	t_{De}=0.25	t_{De}=0.25
Fig 4b: (Rad-CR)	Radial – Constant Rate	t_{De}=0.25	t_{De}=0.25
Fig 4c: (Lin-CP)	Linear – Constant Pressure	t_{De}=0.1	t_{De}=0.25*
Fig 4d: (Lin-CR)	Linear – Constant Rate	t_{De}=0.1	t_{De}=0.5
** tDe=0.50 if t_{MB} is used instead of t*			

Nomenclature:

A	Area
c	Compressibility
k	Permeability
p	Pressure
p_w	Flowing pressure at well
q	Rate
r_e	Exterior radius of reservoir
r_{inv}	Radius of investigation
t	Time (days)
x_e	Exterior boundary
φ	Porosity
μ	Viscosity
p_{wD}	$p_{wD} = \frac{\Delta p\, kh}{141.2 qB\mu}$
t_{DA}	$t_{DA} = \frac{0.00633k\, t(days)}{\emptyset\mu cA}$
t_{De} (Radial)	$t_{De} = \frac{0.00633k\, t(days)}{\emptyset\mu c{r_e}^2}$
t_{De} (Linear)	$t_{De} = \frac{0.00633k\, t(days)}{\emptyset\mu c{x_e}^2}$
t_{MB}	Material Balance Time = Cumulative Production/rate

References:

1. Tabatabaie, H., Pooladi-Darvish, M. and Mattar, L: "Distance of Investigation for Unconventional Reservoirs could be Misleading", SPE-187077, ATCE San Antonio, October 2017
2. Wattenbarger, R. A., El-Banbi, A. H., Villegas, M. E., Maggard, J. B.,. "Production Analysis of Linear Flow into Fractured Tight Gas Wells". SPE-3993,1SPE Rocky Mountain Regional/Low-Permeability Reservoirs Symposium, Denver, April 1998.
3. Behmanesh, H., Clarkson, C. R., Tabatabaie, S. H., Heidari Sureshjani, M.. "Impact of Distance-of-Investigation Calculations on Rate-Transient Analysis of Unconventional Gas and Light-Oil Reservoirs: New Formulations for Linear Flow". JCPT, 54 (6), pp. 509–519, 2015