Rate Transient Analysis

1-4: TRADITIONAL DECLINE ANALYSIS

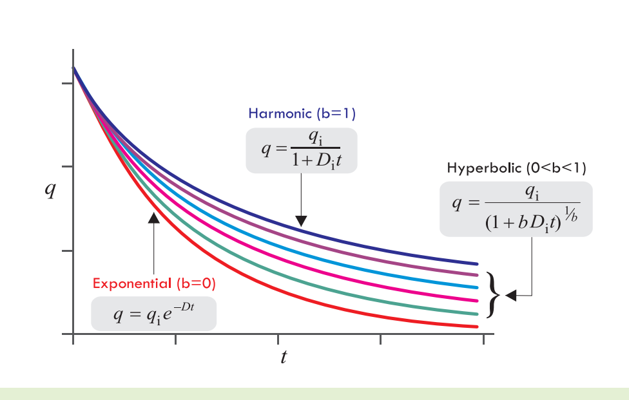
EXPONENTIAL DECLINE:
- Decline rate is constant.
- Log flow rate vs. time is a straight line.
- Flow rate vs. cumulative production is a straight line.
- Provides minimum EUR (Expected Ultimate Recovery).

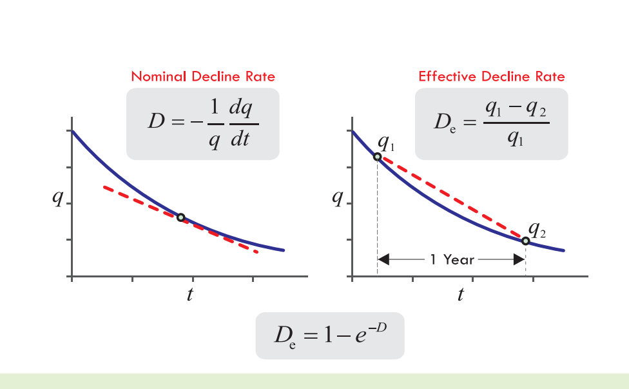
HYPERBOLIC DECLINE:
- Decline rate is not constant ($D=Kq^b$).
- Straight line plots are NOT practical and b is determined by nonlinear curve fit.

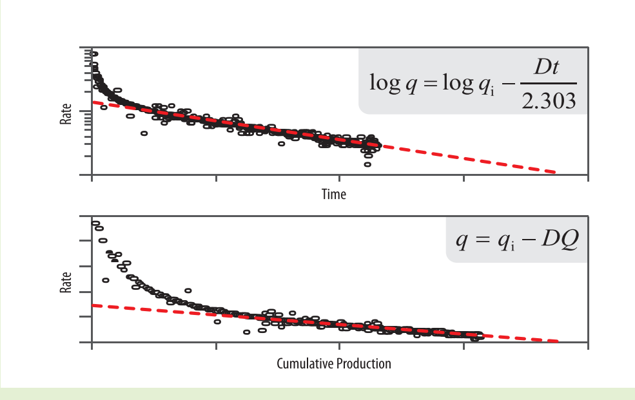
b value	Reservoir Drive Mechanism
0	Single phase liquid (oil above bubble point) Single phase gas at high pressure
0.1-0.4	Solution gas drive
0.4-0.5	Single phase gas
0.5	Effective edge water drive
0.5-1.0	Commingled layered reservoirs

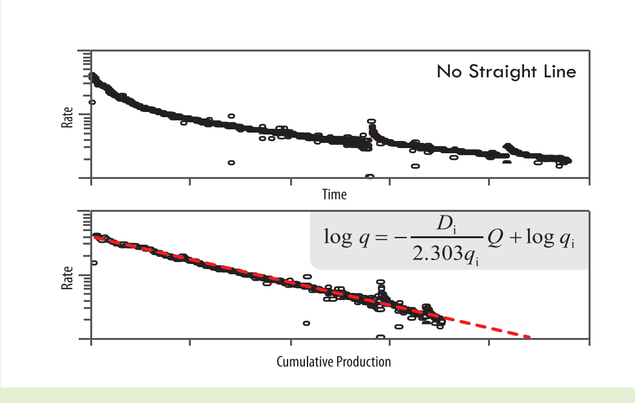
HARMONIC DECLINE:
- Decline rate is directly proportional to flow rate (b=1).
- Log flow rate vs. cumulative production is a straight line.

SUMMARY:
- Boundary-dominated flow only.
- Constant operating conditions.
- Developed using empirical relationships.
- Quick and simple to determine EUR.
- EUR depends on operating conditions.
- Does NOT use pressure data.
- b depends on drive mechanism.


1. Traditional (Arps) Decline Curves


2. Decline Rate Definitions


3. Exponential Decline


4. Harmonic Decline

5-10: FETKOVICH ANALYSIS

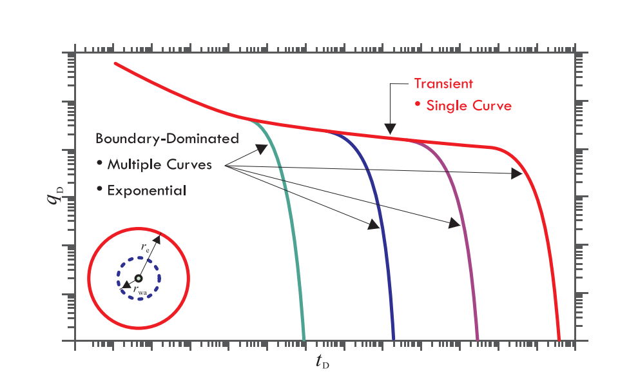
- q_D and t_D definitions are similar to well test.
- Convenient for transient flow.
- Results in single transient stem but multiple boundary-dominated stems.

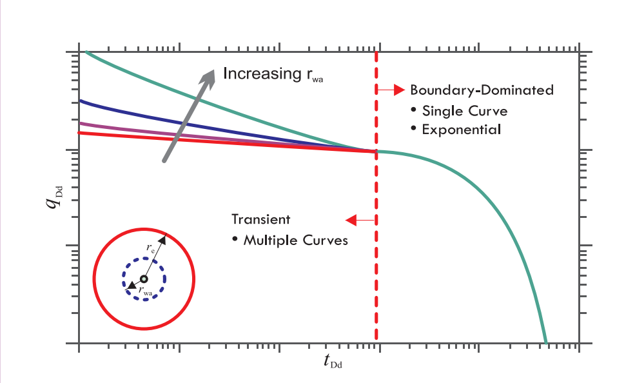
$$q_D=\frac{141.2qB\mu}{kh(p_i-p_{wf})},\quad t_D=\frac{0.00633kt}{\phi\mu c_t r_{wa}^2}$$

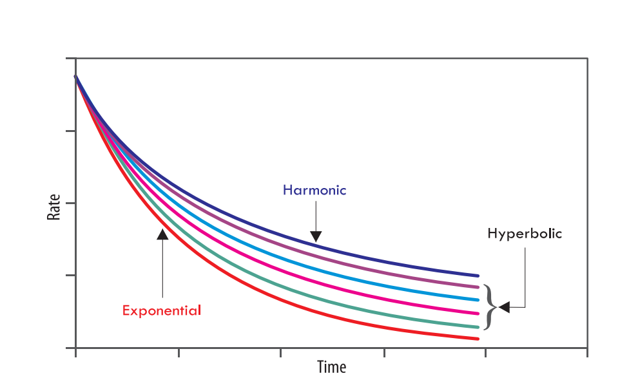
- q_{Dd} and t_{Dd} definitions are convenient for production data analysis.
- Convenient for boundary-dominated flow.
- Results in single boundary-dominated stem but multiple transient stems.

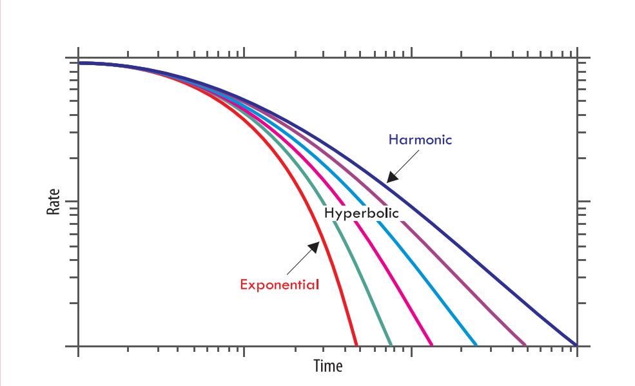
$$q_{Dd}=q_D\ln(r_{eD}-0.5),\quad t_{Dd}=\frac{2t_D}{(\ln r_{eD}-0.5)(r_{eD}^2-1)}$$

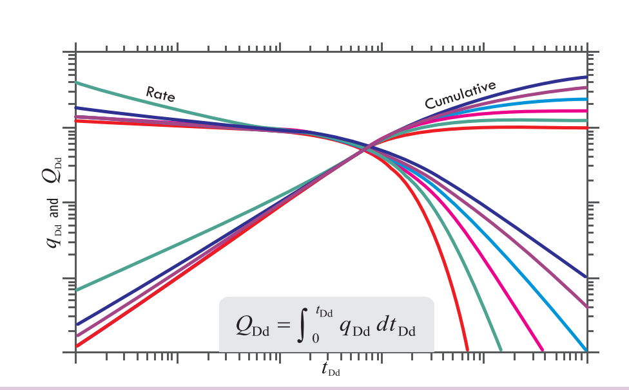
Replot on Log-Log Scale

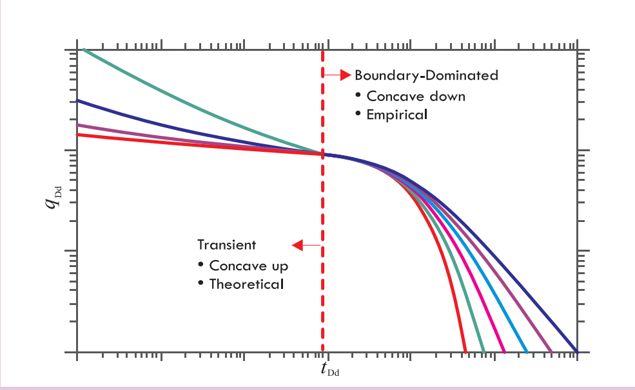

5. Analytical: Constant Flowing Pressure


6. Analytical: Constant Flowing Pressure


7. Empirical: Arps Depletion Stems


8. Empirical: Arps-Fetkovich Depletion Stems


10. Fetkovich/Cumulative Type Curves

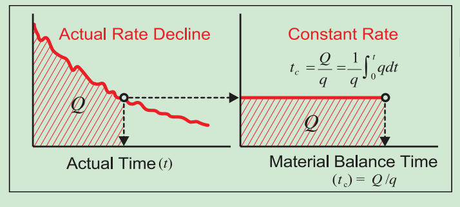

9. Fetkovich Type Curves

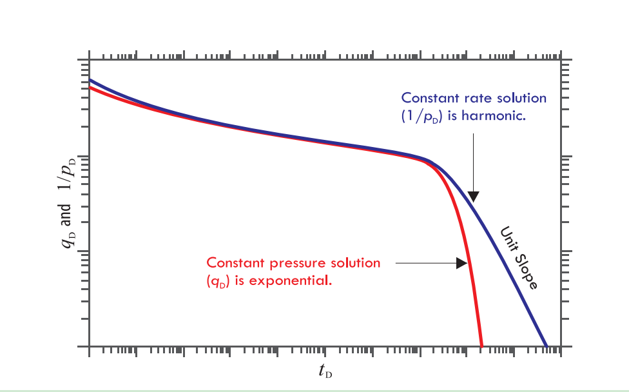
SUMMARY:
- Combines transient with boundary-dominated flow.
- Transient: Analytical, constant pressure solution.
- Boundary-Dominated: Empirical, identical to traditional (Arps).
- Constant operating conditions.
- Used to estimate EUR, skin and permeability.
- EUR depends on operating conditions.
- Does NOT use pressure data.
- Cumulative curves are smoother than rate curves.
- Combined cumulative and rate type curves give more unique match (Figure 10).

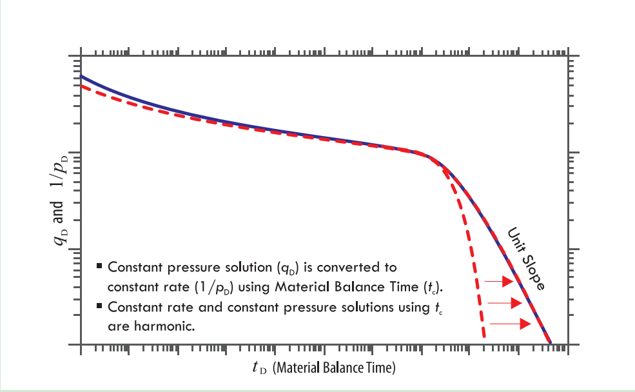
11-14: MODERN DECLINE ANALYSIS: BASIC CONCEPTS

11-12: MATERIAL BALANCE TIME

- Material Balance Time (t_c) effectively converts constant pressure solution to the corresponding constant rate solution.
- Exponential curve plotted using Material Balance Time becomes harmonic.
- Material Balance Time is rigorous during boundary-dominated flow.




11. Comparison of q_D and $1/p_D$


12. Equivalence of q_D and $1/p_D$

- Constant pressure solution (q_D) is converted to constant rate ($1/p_D$) using Material Balance Time (t_c).
- Constant rate and constant pressure solutions using t_c are harmonic.

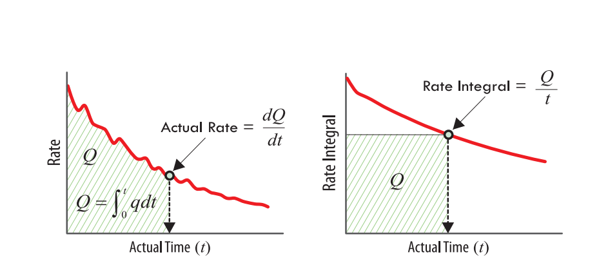
13-14: TYPE CURVE INTERPRETATION AIDS

Rate (Normalized) $\frac{q}{\Delta p}$
- Combines rate with flowing pressure.

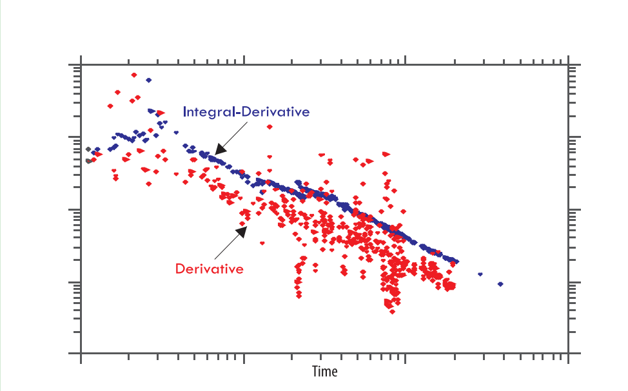
Integral (Normalized Rate) $\frac{1}{t_c}\int_0^{t_c}\frac{q}{\Delta p}dt$
- Smoothes noisy data but attenuates the reservoir signal.

Derivative (Normalized Rate) $-\frac{d(\frac{q}{\Delta p})}{d(\ln t_c)}$
- Amplifies reservoir signal but amplifies noise as well.

Integral-Derivative (Normalized Rate) $-\frac{d}{d(\ln t_c)}(\frac{1}{t_c}\int_0^{t_c}\frac{q}{\Delta p}dt)$
- Smoothes the scatter of the derivative.


13. Concept of Rate Integral

- Rate Integral is cumulative average rate.
- Rate Integral is an effective way to remove noise.


14. Derivative and Integral-Derivative

15-18: GAS FLOW CONSIDERATIONS

15-16: PSEUDO-PRESSURE

Gas properties vary with pressure:
- Z-factor (Pseudo-Pressure, Figures 15 & 16)
- Viscosity (Pseudo-Pressure & Pseudo-Time, Figures 15, 16 & 18)
- Compressibility (Pseudo-Time, Figures 17 & 18)

- Pseudo-pressure corrects for changing viscosity and Z-factor with pressure.
- In all equations for liquid, replace pressure (p) with pseudo-pressure (p_p).

Note: For gas, $q_D=\frac{1}{p_D}=\frac{1.417\times10^6qT}{kh(p_{pi}-p_{pwf})}$

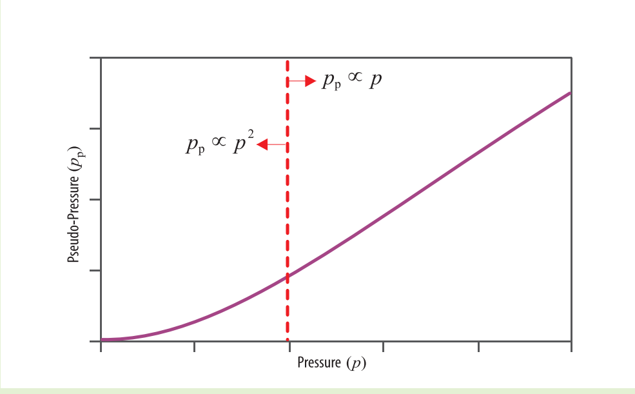
15. Darcy's Law

- Liquid (Constant Viscosity): $\Delta p\propto q$
- Gas: Viscosity and Z-factor are not constant. Define Pseudo-Pressure (p_p):

$$p_p=2\int_{p_0}^{p}\frac{pdp}{\mu_gZ}$$

Pseudo-Pressure (p_p) corrects for changing viscosity (μ_g) and Z-factor with pressure.

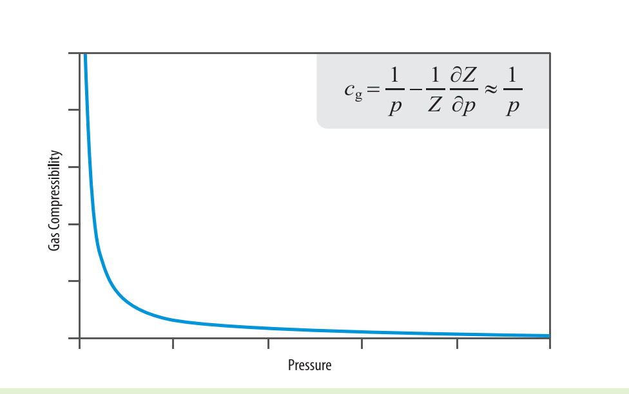
Darcy's Law for Gas: $\Delta p_p\propto q$


16. Pseudo-Pressure (p_p)

17-18: PSEUDO-TIME

- Compressibility represents energy in reservoir.
- Gas compressibility is strong function of pressure (especially at LOW PRESSURES).
- Ignoring compressibility variation can result in significant error in original gas-in-place (G) calculation.
- Pseudo-time(t_a) corrects for changing viscosity and compressibility with pressure.
- Pseudo-time calculation is ITERATIVE because it depends on μ_g and c_t at average reservoir pressure, and average reservoir pressure depends on G (usually known).

Note: Pseudo-time in build-up testing is evaluated at well flowing pressure NOT at average reservoir pressure.


17. Gas Compressibility Variation

18. Pseudo-Time (t_a)

- In all equations for liquid, replace Time (t) with Pseudo-Time (t_a).

$$t_a=(\mu_gc_t)_i\int_0^t\frac{dt}{\bar{\mu}_g\bar{c}_t}$$

- Convert material balance time (t_c) to Material Balance Pseudo-Time (t_{ca}).

$$t_{ca}=\frac{1}{q}\int_0^{t_a}qdt_a=\frac{(\mu_gc_t)_i}{q}\int_0^t\frac{qdt}{\bar{\mu}_g\bar{c}_t}$$

Note: $\bar{\mu}_g$ and $\bar{c}_t$ are evaluated at average reservoir pressure (unlike build-up testing).

19-22: FLOWING MATERIAL BALANCE

Oil

$$p_i-p_{wf}=\frac{N_p}{c_tN}+qb_{pss}$$

$$\frac{q}{p_i-p_{wf}}=-\frac{N_p}{c_tNb_{pss}(p_i-p_{wf})}+\frac{1}{b_{pss}}$$

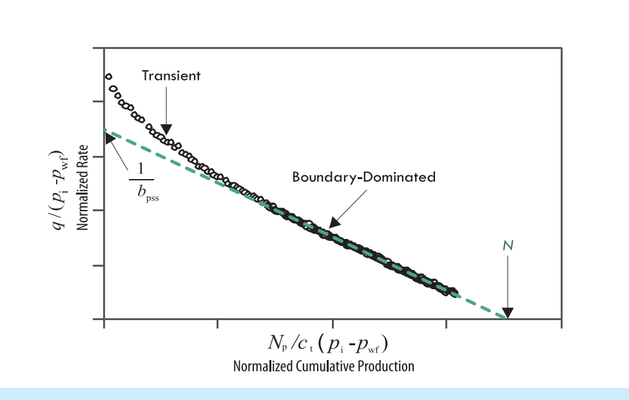
Note: b_{pss} is the inverse of productivity index and is constant during boundary-dominated flow.

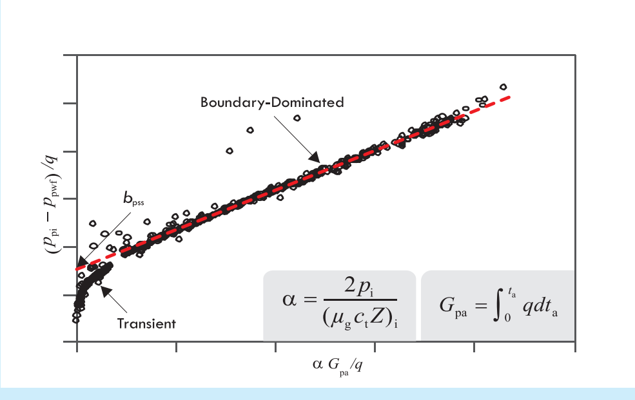
Gas

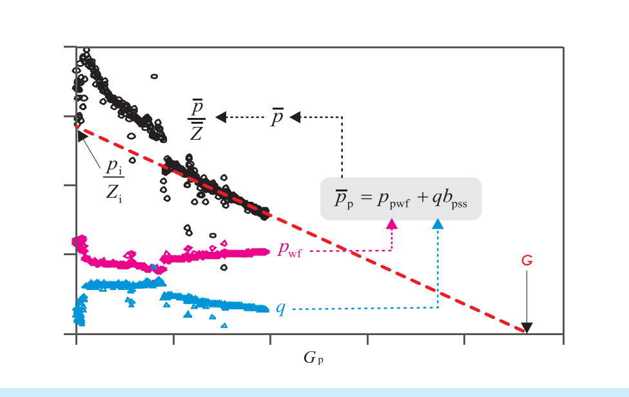
$$p_{pi}-p_{pwf}=\frac{\alpha G_{pa}}{G_i}+qb_{pss}$$

$$\frac{p_{pi}-p_{pwf}}{q}=\frac{\alpha G_{pa}}{qG}+b_{pss}$$

$$\bar{p}_p=p_{pwf}+qb_{pss}$$


19. Oil: Flowing Material Balance


20. Gas: Determination of b_{pss}


21. Gas: Flowing Material Balance

22. Procedure to Calculate Gas-In-Place

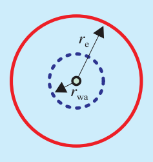
Calculating *G* is ITERATIVE:
1. Estimate G; plot $\bar{p}/\bar{Z}$ vs. G_p from p_i/Z_i to G.
2. At any time, G_p is known. Determine $\bar{p}$ at G_p from $\bar{p}/\bar{Z}$ plot.
3. Obtain $\bar{\mu}_g$ and $\bar{c}_t$ at $\bar{p}$.
4. Convert t to t_a and p_{wf} to p_{pwf} (Figures 18 & 16).
5. Determine b_{pss} from Figure 20.
6. Determine $\bar{p}$ from $\bar{p}_p=p_{pwf}+qb_{pss}$.
7. Plot $\bar{p}/\bar{Z}$ vs. G_p and determine new G.
8. Repeat steps 2-7 until G converges.

SUMMARY:
- Uses flowing data. No shut-in required.
- Applicable to oil and gas.
- Determines hydrocarbon-in-place, N or G.
- Oil (N): Direct calculation.
- Gas (G): Iterative calculation because of pseudo-time.
- Simple yet powerful.
- Data readily available (wellhead pressure can be converted to bottomhole pressure).
- Supplements static material balance.
- Ideal for low permeability reservoirs.

23-32: RADIAL TYPE CURVES

23-24: RADIAL FLOW MODEL: TYPE CURVE ANALYSIS

All radial flow type curves are based on the same reservoir model:
- Well in centre of cylindrical homogeneous reservoir.
- No flow outer boundary.
- Skin factor represented by r_{wa}.
- Information content of all type curves (Figures 25-32) is the same.
- The shapes are different because of different plotting formats.
- Each format represents a different "look" at the data and emphasizes different aspects.



23. Calculations for Oil (Agarwal-Gardner Type Curves)

$$k=\frac{141.2B_o\mu_o}{h}\left(\frac{q/\Delta p}{q_D}\right)_{match}$$

$$r_{wa}=\frac{\sqrt{\frac{0.00633k}{\pi\phi\mu_oc_t}\left(\frac{t_c}{t_{DA}}\right)_{match}}}{(r_{eD})_{match}},\quad s=\ln\left(\frac{r_w}{r_{wa}}\right)$$

$$r_e=\sqrt{\frac{0.00633k}{\pi\phi\mu_oc_t}\left(\frac{t_c}{t_{DA}}\right)_{match}},\quad N=\frac{\pi r_e^2\phi hS_{oi}}{B_{oi}}$$

24. Calculations for Gas (Agarwal-Gardner Type Curves)

$$k=\frac{1.417\times10^6T}{h}\left(\frac{q/\Delta p_p}{q_D}\right)_{match}$$

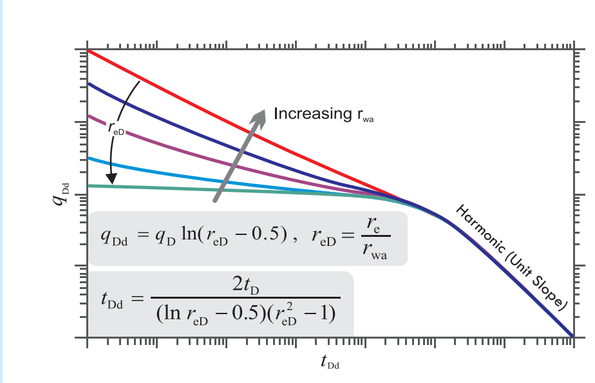
$$r_{wa}=\frac{\sqrt{\frac{0.00633k}{\pi(\phi\mu_gc_t)_i}\left(\frac{t_{ca}}{t_{DA}}\right)_{match}}}{(r_{eD})_{match}},\quad s=\ln\left(\frac{r_w}{r_{wa}}\right)$$

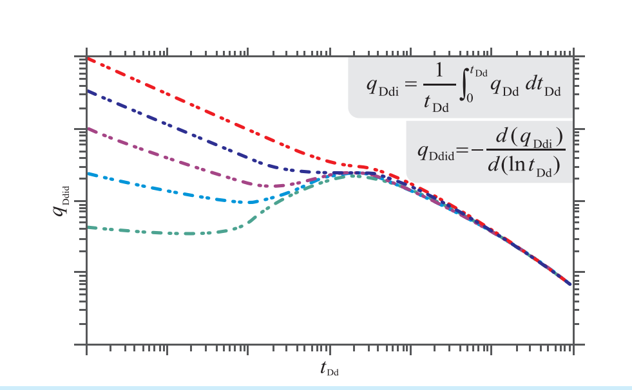
$$r_e=\sqrt{\frac{0.00633k}{\pi(\phi\mu_gc_t)_i}\left(\frac{t_{ca}}{t_{DA}}\right)_{match}},\quad G=\frac{\pi r_e^2\phi hS_{gi}}{B_{gi}}$$

Note: Gas calculations are ITERATIVE because of pseudo-time.

25-26: BLASINGAME

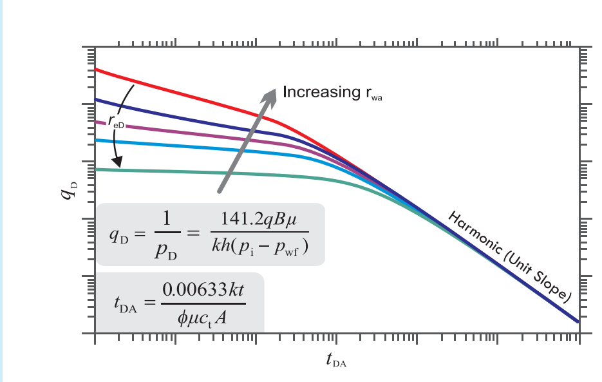
- q_{Dd} and t_{Dd} definitions are similar to Fetkovich.
- Normalized rate ($q/\Delta p$ or $q/\Delta p_p$) is plotted.
- Three sets of type curves:
  1. q_{Dd} vs. t_{Dd} (Figure 25).
  2. Rate integral (q_{Ddi}) vs. t_{Dd} (has the same shape as q_{Dd}).
  3. Rate integral-derivative (q_{Ddid}) vs. t_{Dd} (Figure 26).
- In general: $q_{Dd}=q_Db_{Dpss},\quad t_{Dd}=\frac{2\pi}{b_{Dpss}}t_{DA}$
- b_{Dpss} is a constant for a particular well / reservoir configuration.

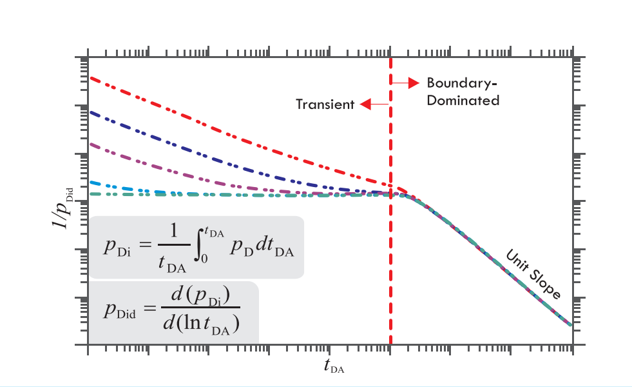

25. Blasingame: Rate (Normalized)


26. Blasingame: Integral-Derivative

27-28: AGARWAL-GARDNER

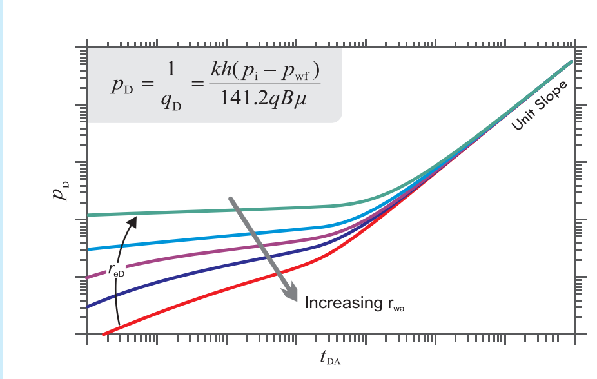
- q_D and t_{DA} definitions are similar to well testing.
- Normalized rate ($q/\Delta p$ or $q/\Delta p_p$) is plotted.
- Three sets of type curves:
  1. q_D vs. t_{DA} (Figure 27).
  2. Inverse of pressure derivative ($1/p_{Dd}$) vs. t_{DA} (not shown).
  3. Inverse of pressure integral-derivative ($1/p_{Did}$) vs. t_{DA} (Figure 28).
- Notes:
  1. Pressure derivative is defined as $p_{Dd}=\frac{d(p_D)}{d(\ln t_{DA})}$
  2. Inverse of pressure derivative is usually too noisy and inverse of pressure integral-derivative is used instead.

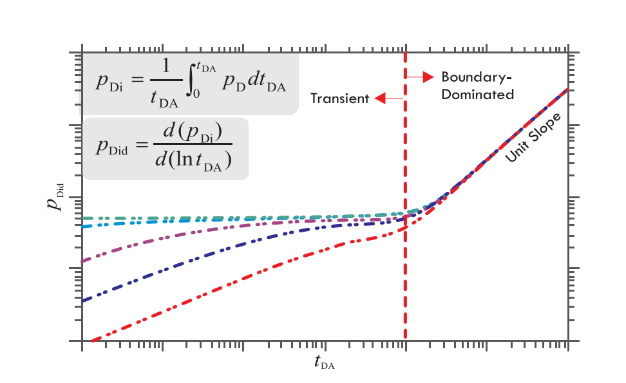

27. Agarwal-Gardner: Rate (Normalized)


28. Agarwal-Gardner: Integral-Derivative

29-30: NORMALIZED PRESSURE INTEGRAL (NPI)

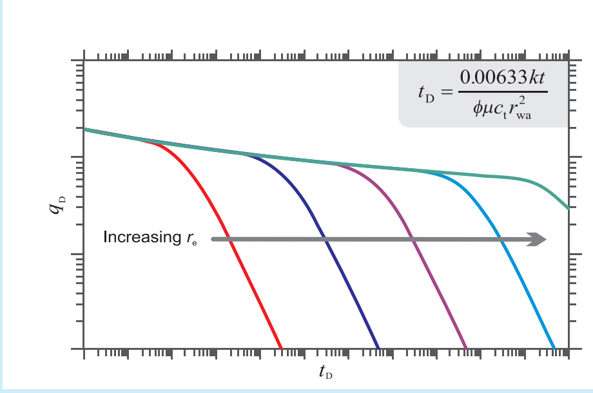
- p_D and t_{DA} definitions are similar to well testing.
- Normalized Pressure ($\Delta p/q$ or $\Delta p_p/q$) is plotted rather than normalized rate ($q/\Delta p$ or $q/\Delta p_p$).
- Three sets of type curves:
  1. p_D vs. t_{DA} (Figure 29).
  2. Pressure integral (p_{Di}) vs. t_{DA} (has the same shape as p_D).
  3. Pressure integral-derivative (p_{Did}) vs. t_{DA} (Figure 30).

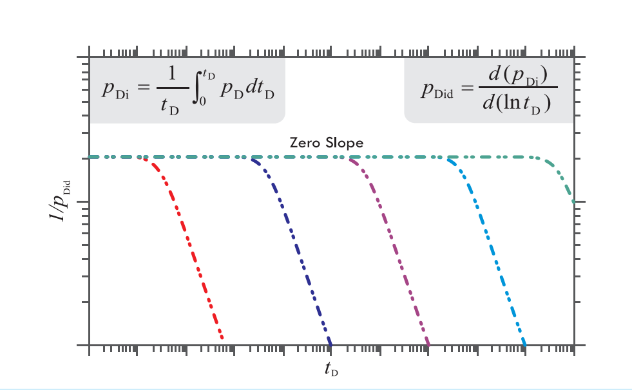

29. NPI: Pressure (Normalized)


30. NPI: Integral-Derivative

31-32: TRANSIENT-DOMINATED DATA

- Similar to Figures 27 & 28 but uses t_D instead of t_{DA}. This format is useful when most of the data are in TRANSIENT flow.
- q_D and t_D definitions are similar to well testing.
- Normalized rate ($q/\Delta p$ or $q/\Delta p_p$) is plotted.
- Three sets of type curves:
  1. q_D vs. t_D (Figure 31).
  2. Inverse of pressure integral ($1/p_{Di}$) vs. t_D (not shown).
  3. Inverse of pressure integral-derivative ($1/p_{Did}$) vs. t_D (Figure 32).

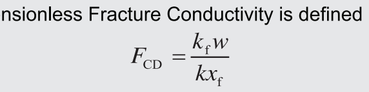

31. Rate (Normalized)


32. Integral-Derivative

33-40: FRACTURE TYPE CURVES

33-37: FINITE CONDUCTIVITY FRACTURE

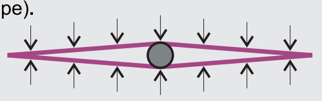
- Fracture with finite conductivity results in bilinear flow (quarter slope).



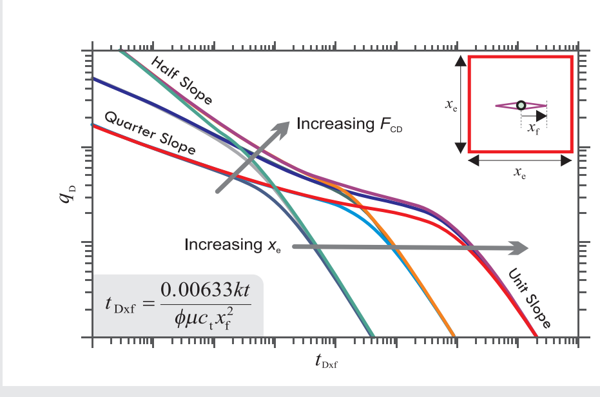
- Dimensionless Fracture Conductivity is defined as:

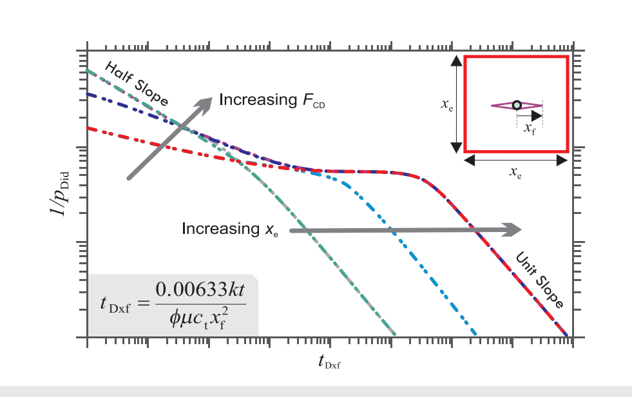
$$F_{CD}=\frac{k_fw}{kx_f}$$

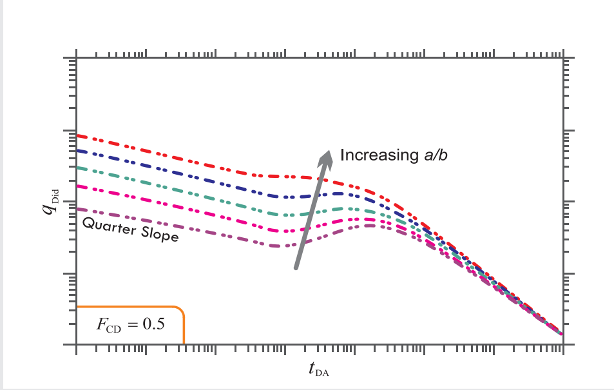
- Fracture with infinite conductivity results in linear flow (half slope).

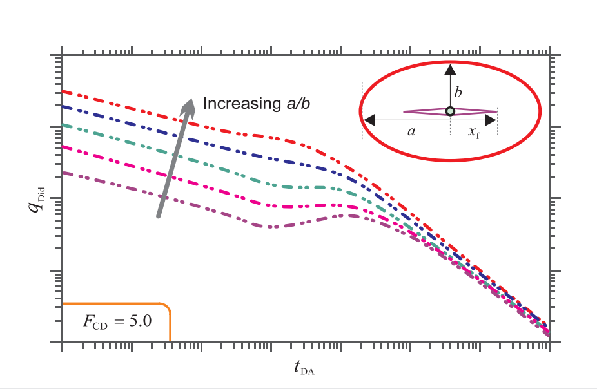


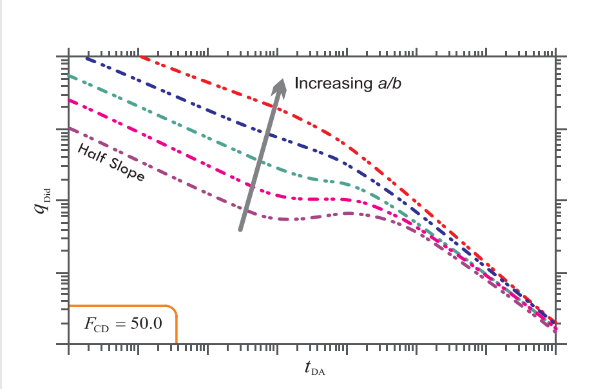
- For F_{CD}>50, the fracture is assumed to have infinite conductivity.


33. Rate

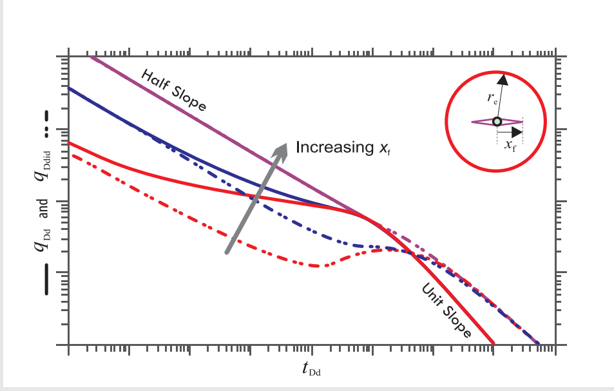

34. Integral-Derivative

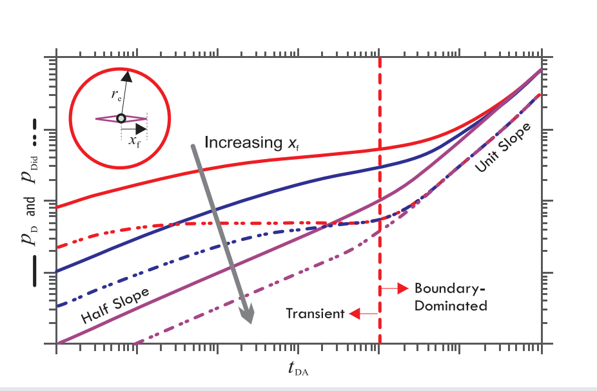

35. Elliptical Flow: Integral-Derivative

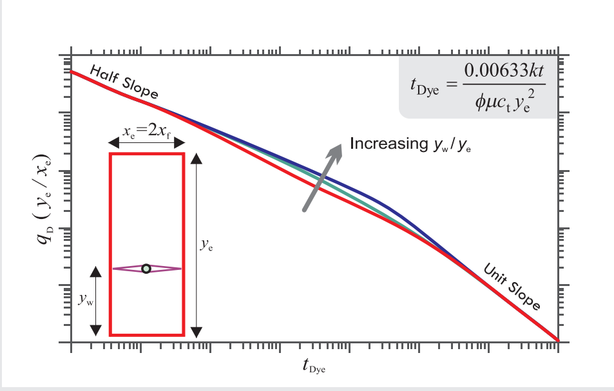

36. Elliptical Flow: Integral-Derivative


37. Elliptical Flow: Integral-Derivative

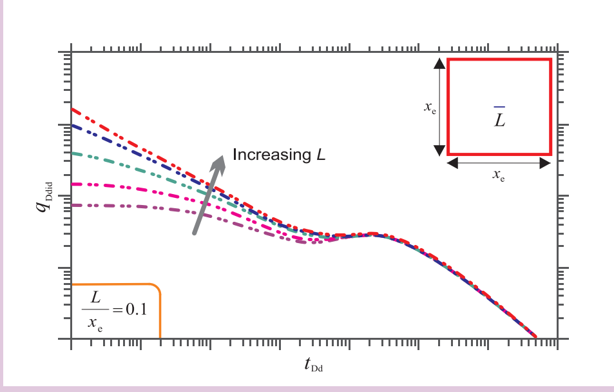
38-40: INFINITE CONDUCTIVITY FRACTURE

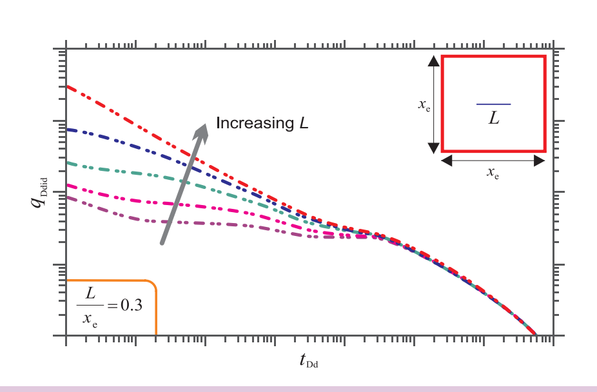

38. Blasingame: Rate and Integral-Derivative

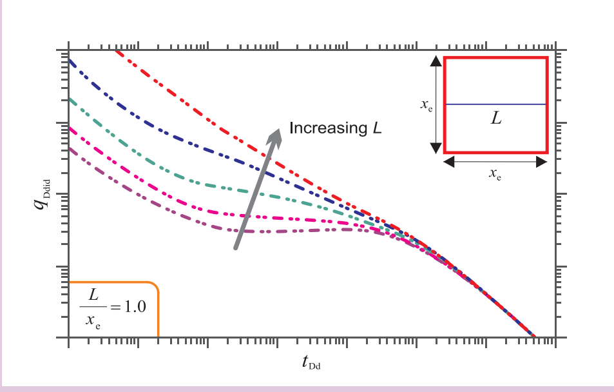

39. NPI: Pressure and Integral-Derivative


40. Wattenbarger: Rate

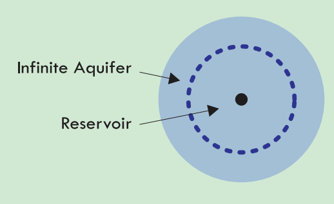
41-43: HORIZONTAL WELL TYPE CURVES


41. Blasingame: Integral-Derivative


42. Blasingame: Integral-Derivative


43. Blasingame: Integral-Derivative

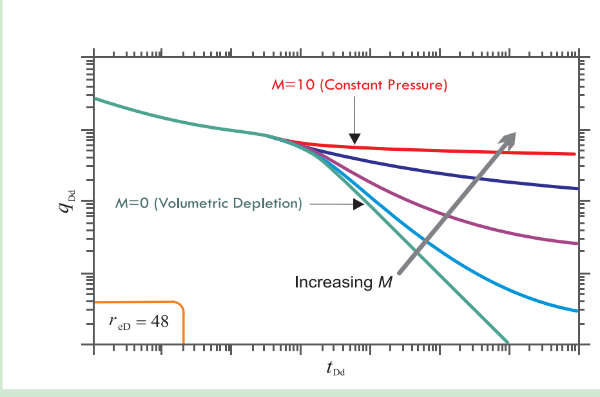
44-45: WATER-DRIVE TYPE CURVES

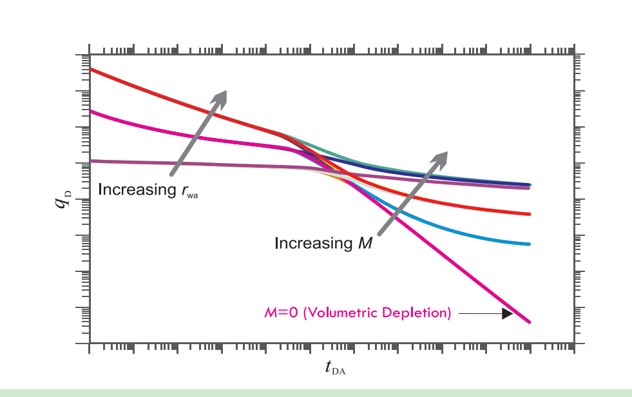


- Mobility ratio (M) represents the strength of the aquifer.

$$M=\frac{k_{aq}}{k_{res}}\frac{\mu_{res}}{\mu_{aq}}$$

- M = 0 is equivalent to Radial Type Curves (Figures 25-32).


44. Blasingame: Rate


45. Agarwal-Gardner: Rate

NOMENCLATURE

a semi-major axis of ellipse
A area
b hyperbolic decline exponent or semi-minor axis of ellipse
b_{Dpss} dimensionless parameter
b_{pss} inverse of productivity index
B formation volume factor
B_{gi} initial gas formation volume factor
B_o oil formation volume factor
B_{oi} initial oil formation volume factor
c_g gas compressibility
c_t total compressibility
$\bar{c}_t$ total compressibility at average reservoir pressure
D nominal decline rate
D_e effective decline rate
D_i initial nominal decline rate
F_{CD} dimensionless fracture conductivity
G original gas-in-place
G_p gas cumulative production
G_{pa} pseudo-cumulative production
h net pay
k permeability
k_{aq} aquifer permeability
k_f fracture permeability
k_h horizontal permeability
k_{res} reservoir permeability
k_v vertical permeability
K constant
L horizontal well length
M mobility ratio
N original oil-in-place
N_p oil cumulative production
p pressure
$\bar{p}$ average reservoir pressure
p_0 reference pressure
p_D dimensionless pressure
p_{Dd} dimensionless pressure derivative
p_{Di} dimensionless pressure integral
p_{Did} dimensionless pressure integral-derivative
p_i initial reservoir pressure
p_p pseudo-pressure
$\bar{p}_p$ pseudo-pressure at average reservoir pressure
p_{pi} initial pseudo-pressure
p_{pwf} pseudo-pressure at well flowing pressure
p_{wf} well flowing pressure
q flow rate
q_D dimensionless rate
q_{Dd} dimensionless rate
q_{Ddi} dimensionless rate integral
q_{Ddid} dimensionless rate integral-derivative
q_i initial flow rate
Q cumulative production
Q_{Dd} dimensionless cumulative production
r_e exterior radius of reservoir
r_{eD} dimensionless exterior radius of reservoir
r_w wellbore radius
r_{wa} apparent wellbore radius
s skin
S_{gi} initial gas saturation
S_{oi} initial oil saturation
t flow time
t_a pseudo-time
t_c material balance time
t_{ca} material balance pseudo-time
t_D dimensionless time
t_{DA} dimensionless time
t_{Dd} dimensionless time
t_{Dxf} dimensionless time
t_{Dye} dimensionless time
T reservoir temperature
w fracture width
x_e reservoir length
x_f fracture half length
y_e reservoir width
y_w well location in y-direction
Z gas deviation factor
$\bar{Z}$ gas deviation factor at average reservoir pressure
Z_i initial gas deviation factor
α constant
ϕ porosity
μ viscosity
μ_{aq} aquifer fluid viscosity
μ_g gas viscosity
$\bar{\mu}_g$ gas viscosity at average reservoir pressure
μ_o oil viscosity
μ_{res} reservoir fluid viscosity

Oil field units; q_g (MMSCFD); t (days)

All analyses described can be performed using IHS Markit's Rate Transient Analysis software RTA.

3923-JH-0117