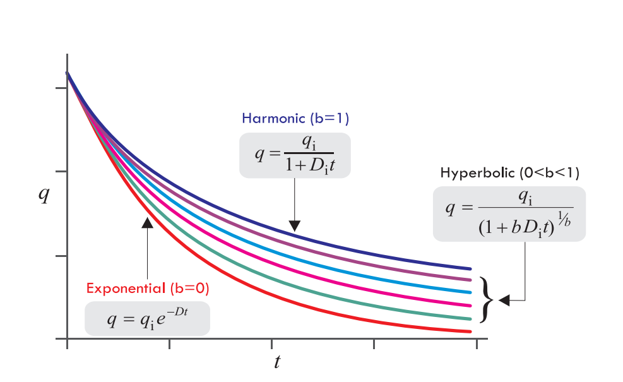
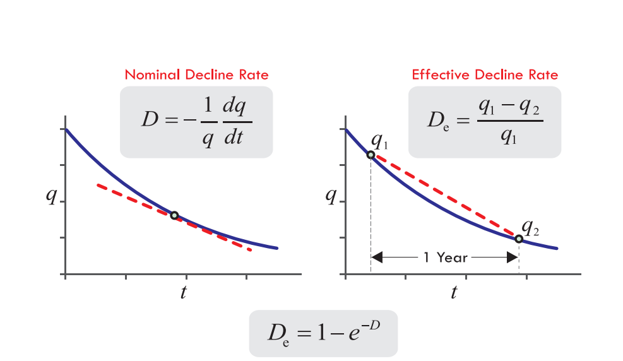
Rate Transient Analysis

1. Traditional (Arps) Decline Curves



2. Decline Rate Definitions



1-4: TRADITIONAL DECLINE ANALYSIS

EXPONENTIAL DECLINE:

- Decline rate is constant.
- Log flow rate vs. time is a straight line.
- Flow rate vs. cumulative production is a straight line.
- Provides minimum EUR (Expected Ultimate Recovery).

HYPERBOLIC DECLINE:

- Decline rate is not constant ($D=Kq^b$).
- Straight line plots are NOT practical and b is determined by nonlinear curve fit.

b value	Reservoir Drive Mechanism
0	Single phase liquid (oil above bubble point) Single phase gas at high pressure
0.1-0.4	Solution gas drive
0.4-0.5	Single phase gas
0.5	Effective edge water drive
0.5-1.0	Commingled layered reservoirs

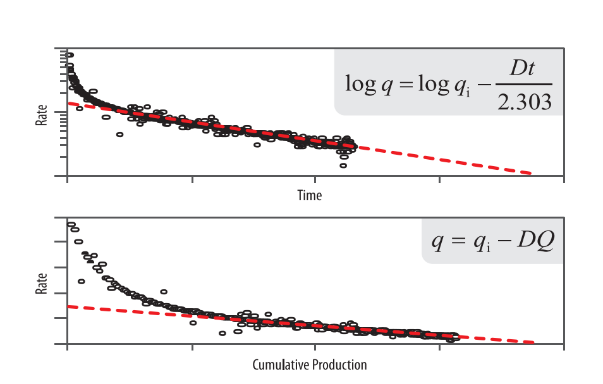
HARMONIC DECLINE:

- Decline rate is directly proportional to flow rate (b=1).
- Log flow rate vs. cumulative production is a straight line.

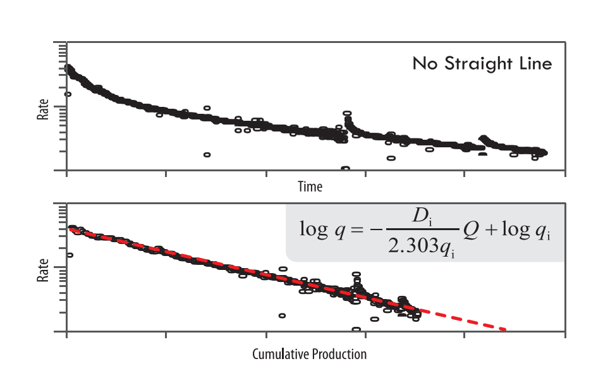
SUMMARY:

- Boundary-dominated flow only.
- Constant operating conditions.
- Developed using empirical relationships.
- Quick and simple to determine EUR.
- EUR depends on operating conditions.
- Does NOT use pressure data.
- b depends on drive mechanism.

3. Exponential Decline

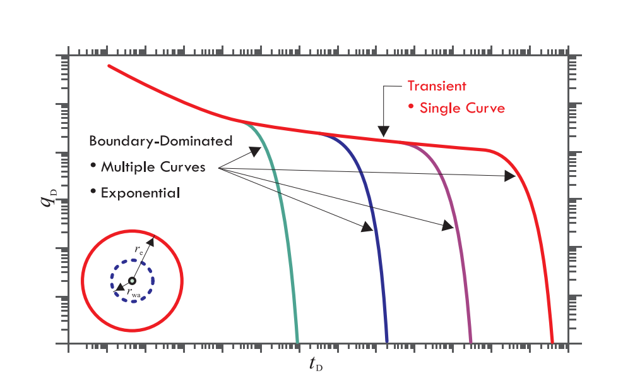


4. Harmonic Decline



5-10: FETKOVICH ANALYSIS

5. Analytical: Constant Flowing Pressure



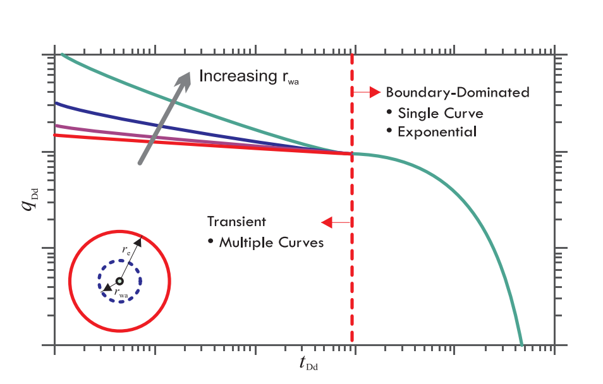
- q_D and t_D definitions are similar to well test.
- Convenient for transient flow.
- Results in single transient stem but multiple boundary-dominated stems.

$$q_D=\frac{141.2qB\mu}{kh(p_i-p_{wf})},\quad t_D=\frac{0.00633kt}{\phi\mu c_t r_{wa}^2}$$

- q_{Dd} and t_{Dd} definitions are convenient for production data analysis.
- Convenient for boundary-dominated flow.
- Results in single boundary-dominated stem but multiple transient stems.

$$q_{Dd}=q_D\ln(r_{eD}-0.5)\,,\ t_{Dd}=\frac{2t_D}{(\ln r_{eD}-0.5)(r_{eD}^2-1)}$$

6. Analytical: Constant Flowing Pressure



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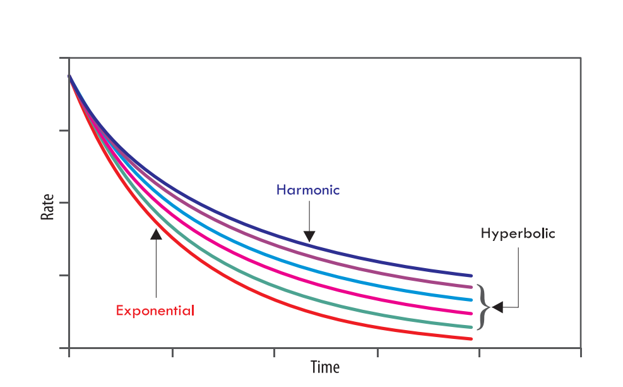
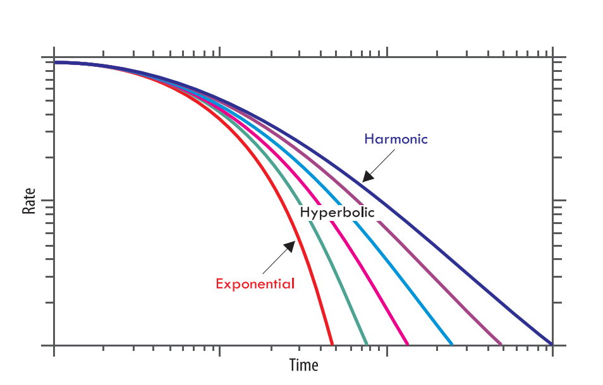
7. Empirical: Arps Depletion Stems



Replot on Log-Log Scale

8. Empirical: Arps-Fetkovich Depletion Stems



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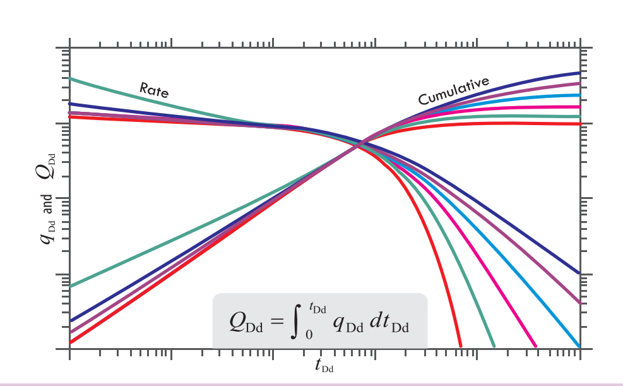
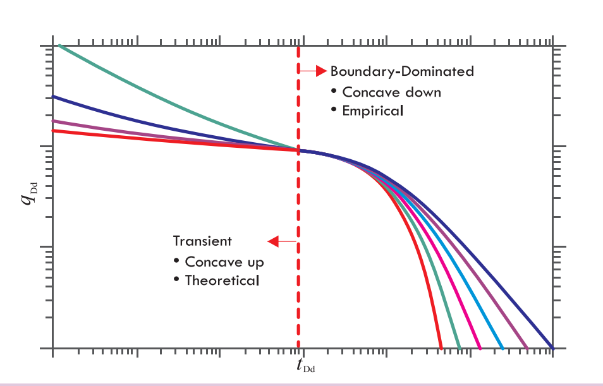
10. Fetkovich/Cumulative Type Curves



SUMMARY:

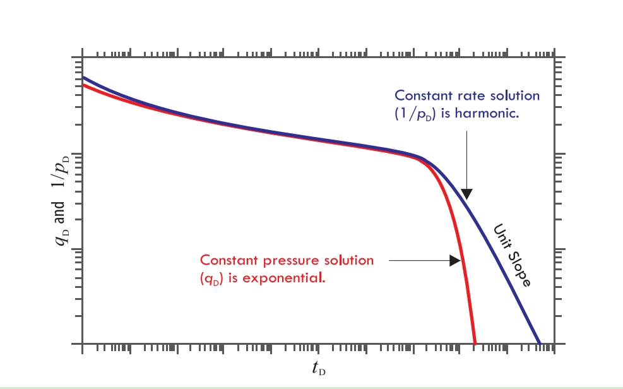
- Combines transient with boundary-dominated flow.
- Transient: Analytical, constant pressure solution.
- Boundary-Dominated: Empirical, identical to traditional (Arps).
- Constant operating conditions.
- Used to estimate EUR, skin and permeability.
- EUR depends on operating conditions.
- Does NOT use pressure data.
- Cumulative curves are smoother than rate curves.
- Combined cumulative and rate type curves give more unique match (Figure 10).

9. Fetkovich Type Curves



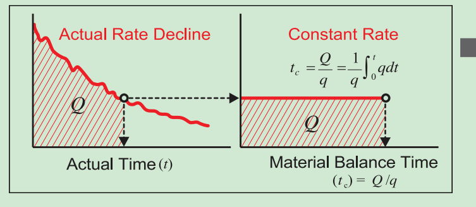
11-14: MODERN DECLINE ANALYSIS: BASIC CONCEPTS

11. Comparison of q_D and $1/p_D$

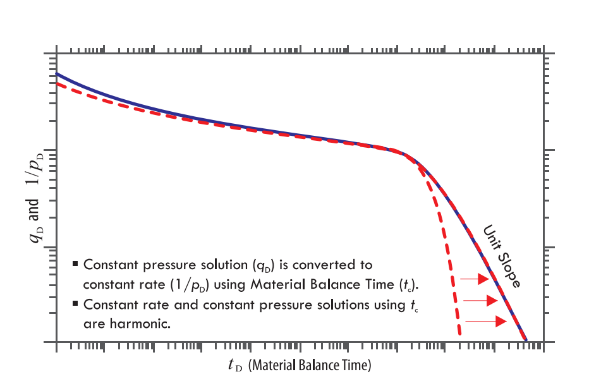


11-12: MATERIAL BALANCE TIME

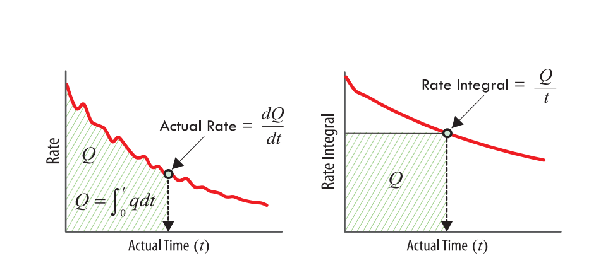
- Material Balance Time (t_c) effectively converts constant pressure solution to the corresponding constant rate solution.
- Exponential curve plotted using Material Balance Time becomes harmonic.
- Material Balance Time is rigorous during boundary-dominated flow.



12. Equivalence of q_D and $1/p_D$



13. Concept of Rate Integral



- Rate Integral is cumulative average rate.
- Rate Integral is an effective way to remove noise.

13-14: TYPE CURVE INTERPRETATION AIDS

Rate (Normalized)

- Combines rate with flowing pressure. $\frac{q}{\Delta p}$

Integral (Normalized Rate)

- Smoothes noisy data but attenuates the reservoir signal. $\frac{1}{t_c}\int_0^{t_c}\frac{q}{\Delta p}dt$

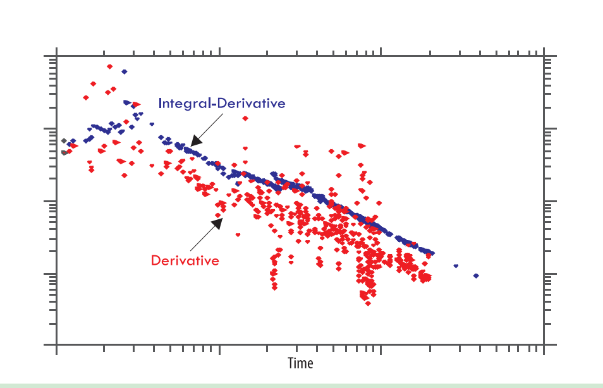
Derivative (Normalized Rate)

- Amplifies reservoir signal but amplifies noise as well. $-\frac{d(\frac{q}{\Delta p})}{d(\ln t_c)}$

Integral-Derivative (Normalized Rate)

- Smoothes the scatter of the derivative. $-\frac{d}{d(\ln t_c)}(\frac{1}{t_c}\int_0^{t_c}\frac{q}{\Delta p}dt)$

14. Derivative and Integral-Derivative



15-18: GAS FLOW CONSIDERATIONS

15. Darcy's Law

- Liquid (Constant Viscosity): $\Delta p\propto q$
- Gas: Viscosity and Z-factor are not constant. Define Pseudo-Pressure (p_p):

$$p_p=2\int_{p_0}^{p}\frac{pdp}{\mu_g Z}$$

Pseudo-Pressure (p_p) corrects for changing viscosity (μ_g) and Z-factor with pressure.

Darcy's Law for Gas: $\Delta p_p\propto q$

15-16: PSEUDO-PRESSURE

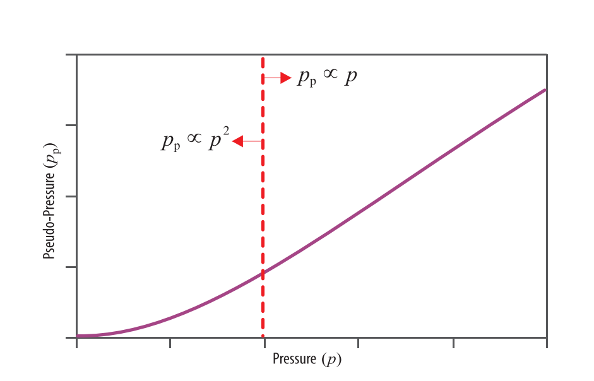
Gas properties vary with pressure:

- Z-factor (Pseudo-Pressure, Figures 15 & 16)
- Viscosity (Pseudo-Pressure & Pseudo-Time, Figures 15, 16 & 18)
- Compressibility (Pseudo-Time, Figures 17 & 18)

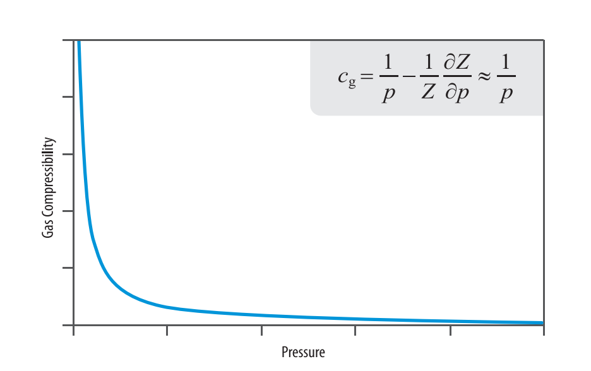
- Pseudo-pressure corrects for changing viscosity and Z-factor with pressure.
- In all equations for liquid, replace pressure (p) with pseudo-pressure (p_p).

Note: For gas, $q_D=\frac{1}{p_D}=\frac{1.417\times10^6qT}{kh(p_{pi}-p_{pwf})}$

16. Pseudo-Pressure (p_p)



17. Gas Compressibility Variation



17-18: PSEUDO-TIME

- Compressibility represents energy in reservoir.
- Gas compressibility is strong function of pressure (especially at LOW PRESSURES).
- Ignoring compressibility variation can result in significant error in original gas-in-place (G) calculation.
- Pseudo-time(t_a) corrects for changing viscosity and compressibility with pressure.
- Pseudo-time calculation is ITERATIVE because it depends on μ_g and c_t at average reservoir pressure, and average reservoir pressure depends on G (usually known).

Note: Pseudo-time in build-up testing is evaluated at well flowing pressure NOT at average reservoir pressure.

18. Pseudo-Time (t_a)

- In all equations for liquid, replace Time (t) with Pseudo-Time (t_a).

$$t_a=(\mu_g c_t)_i\int_0^t\frac{dt}{\bar{\mu}_g\bar{c}_t}$$

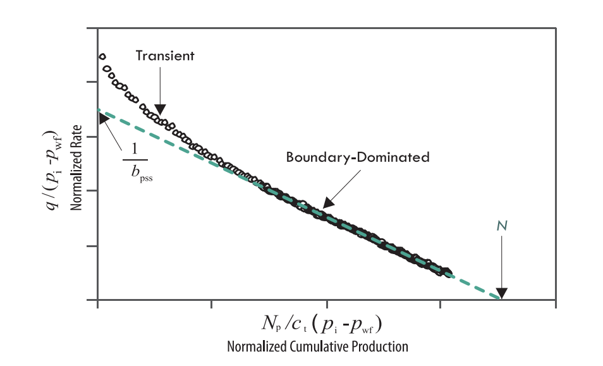
- Convert material balance time (t_c) to Material Balance Pseudo-Time (t_{ca}).

$$t_{ca}=\frac{1}{q}\int_0^{t_a}qdt_a=\frac{(\mu_g c_t)_i}{q}\int_0^t\frac{qdt}{\bar{\mu}_g\bar{c}_t}$$

Note: $\bar{\mu}_g$ and $\bar{c}_t$ are evaluated at average reservoir pressure (unlike build-up testing).

19-22: FLOWING MATERIAL BALANCE

19. Oil: Flowing Material Balance



Oil

$$p_i-p_{wf}=\frac{N_p}{c_tN}+qb_{pss}$$

$$\frac{q}{p_i-p_{wf}}=-\frac{N_p}{c_tNb_{pss}(p_i-p_{wf})}+\frac{1}{b_{pss}}$$

Note: b_{pss} is the inverse of productivity index and is constant during boundary-dominated flow.

Gas

$$p_{pi}-p_{pwf}=\frac{\alpha G_{pa}}{G_i}+qb_{pss}$$

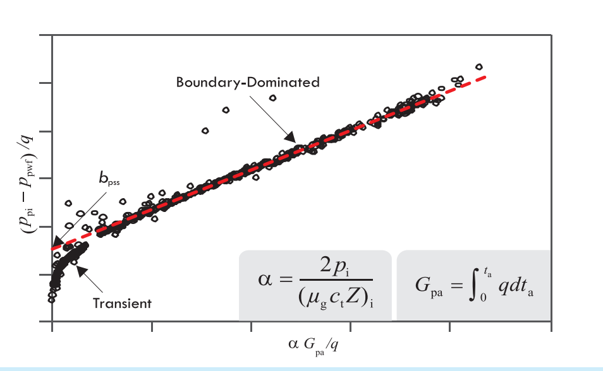
$$\frac{p_{pi}-p_{pwf}}{q}=\frac{\alpha G_{pa}}{qG}+b_{pss}$$

$$\bar{p}_p=p_{pwf}+qb_{pss}$$

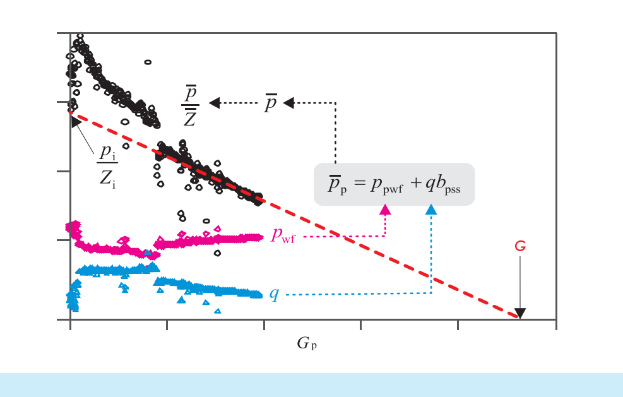
SUMMARY:

- Uses flowing data. No shut-in required.
- Applicable to oil and gas.
- Determines hydrocarbon-in-place, N or G.
- Oil (N): Direct calculation.
- Gas (G): Iterative calculation because of pseudo-time.
- Simple yet powerful.
- Data readily available (wellhead pressure can be converted to bottomhole pressure).
- Supplements static material balance.
- Ideal for low permeability reservoirs.

20. Gas: Determination of b_{pss}



21. Gas: Flowing Material Balance



22. Procedure to Calculate Gas-In-Place

Calculating G is ITERATIVE:

1. Estimate G; plot $\bar{p}/\bar{Z}$ vs. G_p from p_i/Z_i to G.
2. At any time, G_p is known. Determine $\bar{p}$ at G_p from $\bar{p}/\bar{Z}$ plot.
3. Obtain $\bar{\mu}_g$ and $\bar{c}_t$ at $\bar{p}$.
4. Convert t to t_a and p_{wf} to p_{pwf} (Figures 18 & 16).
5. Determine b_{pss} from Figure 20.
6. Determine $\bar{p}$ from $\bar{p}_p=p_{pwf}+qb_{pss}$.
7. Plot $\bar{p}/\bar{Z}$ vs. G_p and determine new G.
8. Repeat steps 2-7 until G converges.

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23-32: RADIAL TYPE CURVES

23. Calculations for Oil (Agarwal-Gardner Type Curves)

$$k=\frac{141.2B_o\mu_o}{h}\left(\frac{q/\Delta p}{q_D}\right)_{match}$$

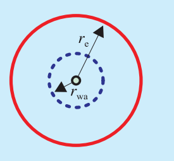
$$r_{wa}=\frac{\sqrt{\frac{0.00633k}{\pi\phi\mu_oc_t}\left(\frac{t_c}{t_{DA}}\right)_{match}}}{(r_{eD})_{match}},\quad s=\ln\left(\frac{r_w}{r_{wa}}\right)$$

$$r_e=\sqrt{\frac{0.00633k}{\pi\phi\mu_oc_t}\left(\frac{t_c}{t_{DA}}\right)_{match}},\quad N=\frac{\pi r_e^2\phi hS_{oi}}{B_{oi}}$$

23-24: RADIAL FLOW MODEL: TYPE CURVE ANALYSIS

All radial flow type curves are based on the same reservoir model:

- Well in centre of cylindrical homogeneous reservoir.
- No flow outer boundary.
- Skin factor represented by r_{wa}.
- Information content of all type curves (Figures 25-32) is the same.
- The shapes are different because of different plotting formats.
- Each format represents a different "look" at the data and emphasizes different aspects.



24. Calculations for Gas (Agarwal-Gardner Type Curves)

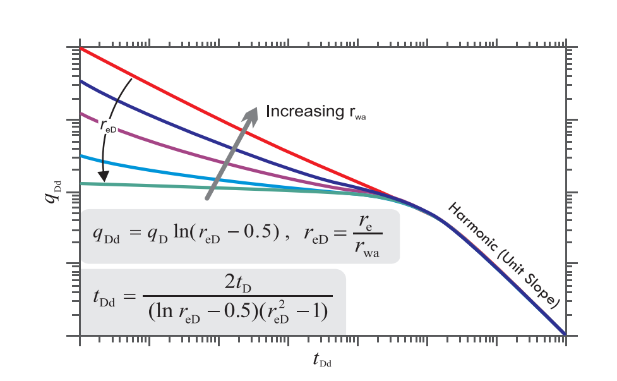
$$k=\frac{1.417\times10^6T}{h}\left(\frac{q/\Delta p_p}{q_D}\right)_{match}$$

$$r_{wa}=\frac{\sqrt{\frac{0.00633k}{\pi(\phi\mu_gc_t)_i}\left(\frac{t_{ca}}{t_{DA}}\right)_{match}}}{(r_{eD})_{match}},\quad s=\ln\left(\frac{r_w}{r_{wa}}\right)$$

$$r_e=\sqrt{\frac{0.00633k}{\pi(\phi\mu_gc_t)_i}\left(\frac{t_{ca}}{t_{DA}}\right)_{match}},\quad G=\frac{\pi r_e^2\phi hS_{gi}}{B_{gi}}$$

Note: Gas calculations are ITERATIVE because of pseudo-time.

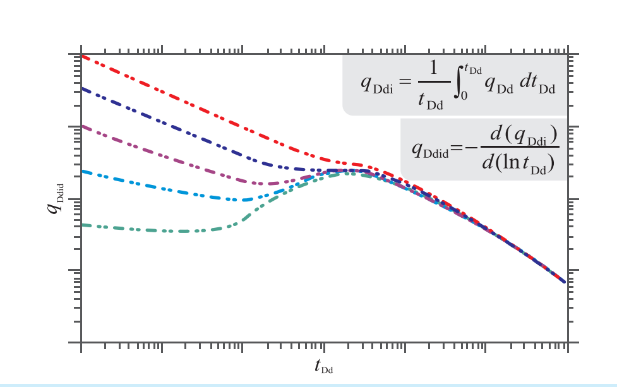
25. Blasingame: Rate (Normalized)



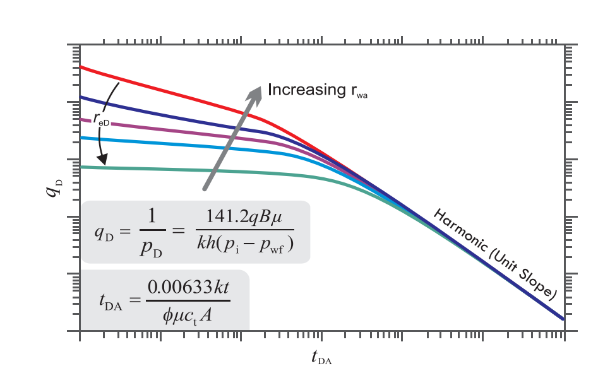
25-26: BLASINGAME

- q_{Dd} and t_{Dd} definitions are similar to Fetkovich.
- Normalized rate ($q/\Delta p$ or $q/\Delta p_p$) is plotted.
- Three sets of type curves:
  1. q_{Dd} vs. t_{Dd} (Figure 25).
  2. Rate integral (q_{Ddi}) vs. t_{Dd} (has the same shape as q_{Dd}).
  3. Rate integral-derivative (q_{Ddid}) vs. t_{Dd} (Figure 26).
- In general: $q_{Dd}=q_Db_{Dpss}\,,\ t_{Dd}=\frac{2\pi}{b_{Dpss}}t_{DA}$
- b_{Dpss} is a constant for a particular well / reservoir configuration.

26. Blasingame: Integral-Derivative



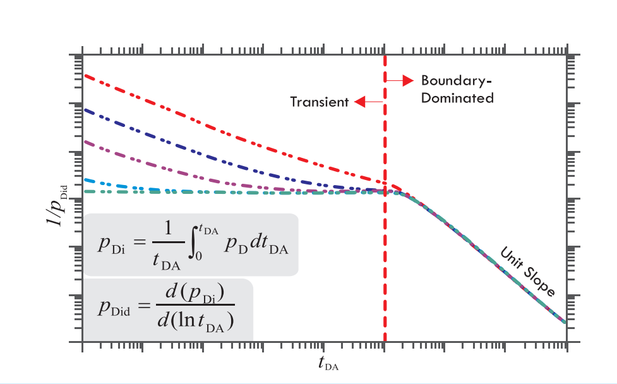
27. Agarwal-Gardner: Rate (Normalized)



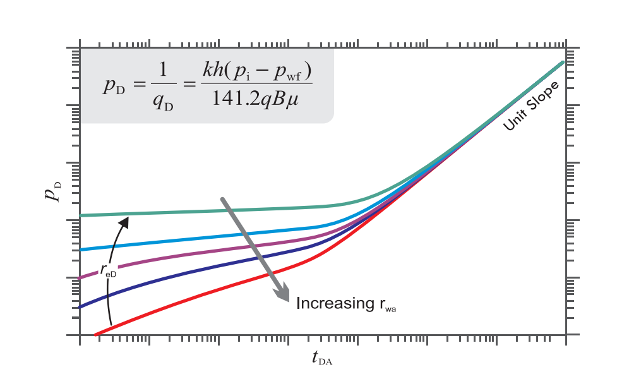
27-28: AGARWAL-GARDNER

- q_D and t_{DA} definitions are similar to well testing.
- Normalized rate ($q/\Delta p$ or $q/\Delta p_p$) is plotted.
- Three sets of type curves:
  1. q_D vs. t_{DA} (Figure 27).
  2. Inverse of pressure derivative ($1/p_{Dd}$) vs. t_{DA} (not shown).
  3. Inverse of pressure integral-derivative ($1/p_{Did}$) vs. t_{DA} (Figure 28).
- Notes:
  1. Pressure derivative is defined as $p_{Dd}=\frac{d(p_D)}{d(\ln t_{DA})}$
  2. Inverse of pressure derivative is usually too noisy and inverse of pressure integral-derivative is used instead.

28. Agarwal-Gardner: Integral-Derivative



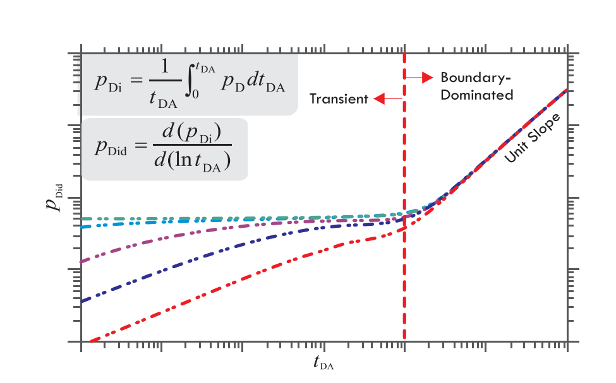
29. NPI: Pressure (Normalized)



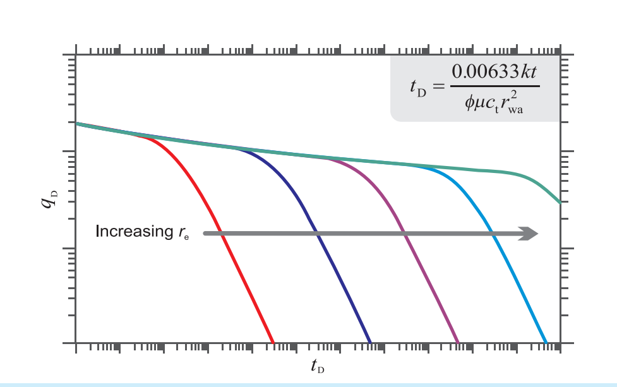
29-30: NORMALIZED PRESSURE INTEGRAL (NPI)

- p_D and t_{DA} definitions are similar to well testing.
- Normalized Pressure ($\Delta p/q$ or $\Delta p_p/q$) is plotted rather than normalized rate ($q/\Delta p$ or $q/\Delta p_p$).
- Three sets of type curves:
  1. p_D vs. t_{DA} (Figure 29).
  2. Pressure integral (p_{Di}) vs. t_{DA} (has the same shape as p_D).
  3. Pressure integral-derivative (p_{Did}) vs. t_{DA} (Figure 30).

30. NPI: Integral-Derivative



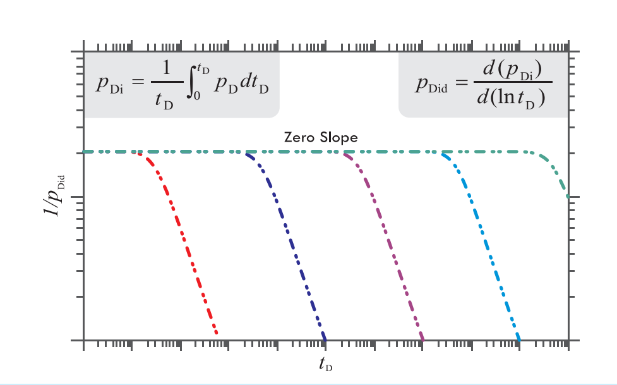
31. Rate (Normalized)



31-32: TRANSIENT-DOMINATED DATA

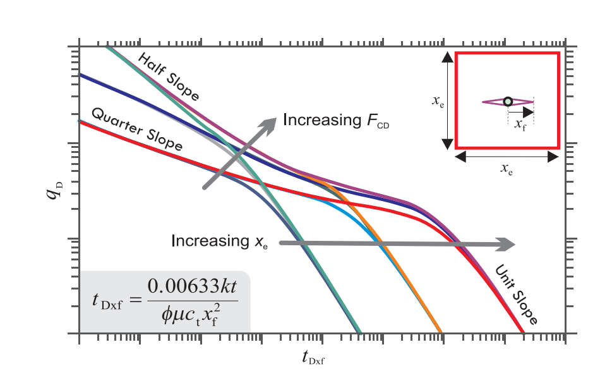
- Similar to Figures 27 & 28 but uses t_D instead of t_{DA}. This format is useful when most of the data are in TRANSIENT flow.
- q_D and t_D definitions are similar to well testing.
- Normalized rate ($q/\Delta p$ or $q/\Delta p_p$) is plotted.
- Three sets of type curves:
  1. q_D vs. t_D (Figure 31).
  2. Inverse of pressure integral ($1/p_{Di}$) vs. t_D (not shown).
  3. Inverse of pressure integral-derivative ($1/p_{Did}$) vs. t_D (Figure 32).

32. Integral-Derivative



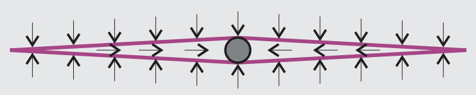
33-40: FRACTURE TYPE CURVES

33. Rate



33-37: FINITE CONDUCTIVITY FRACTURE

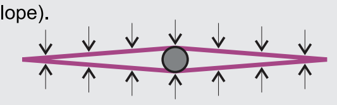
- Fracture with finite conductivity results in bilinear flow (quarter slope).



- Dimensionless Fracture Conductivity is defined as:

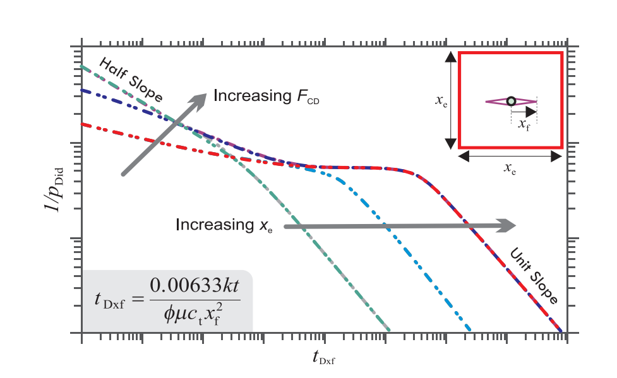
$$F_{CD}=\frac{k_fw}{kx_f}$$

- Fracture with infinite conductivity results in linear flow (half slope).

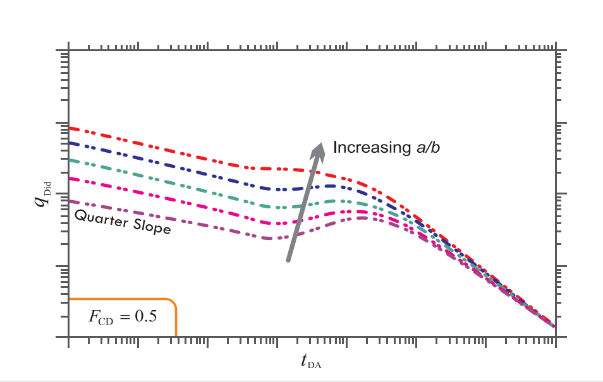


- For F_{CD}>50, the fracture is assumed to have infinite conductivity.

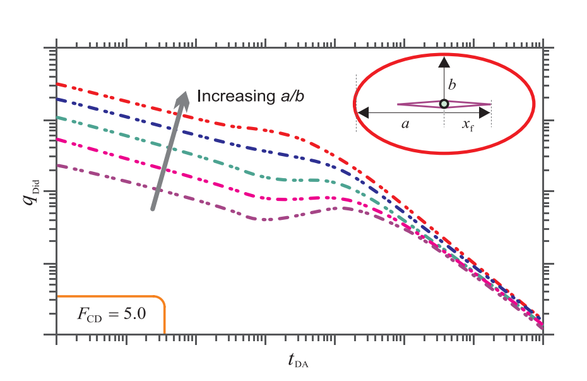
34. Integral-Derivative



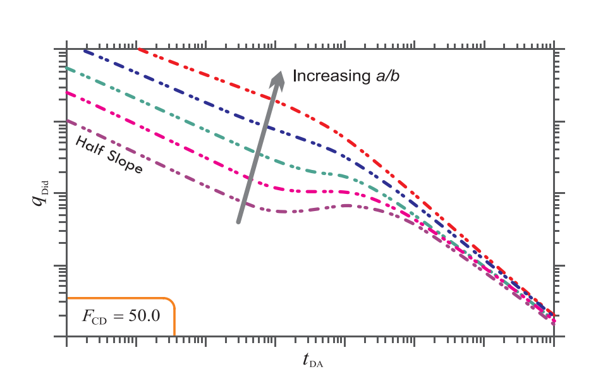
35. Elliptical Flow: Integral-Derivative



36. Elliptical Flow: Integral-Derivative

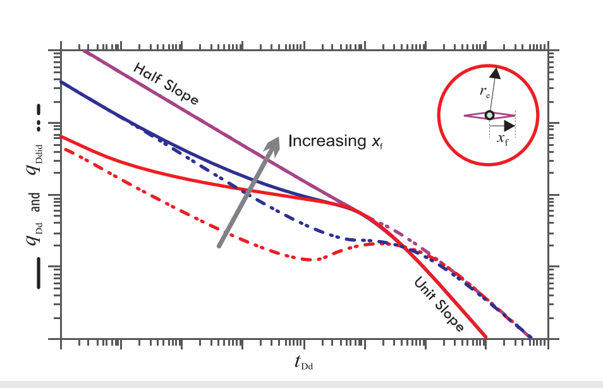


37. Elliptical Flow: Integral-Derivative

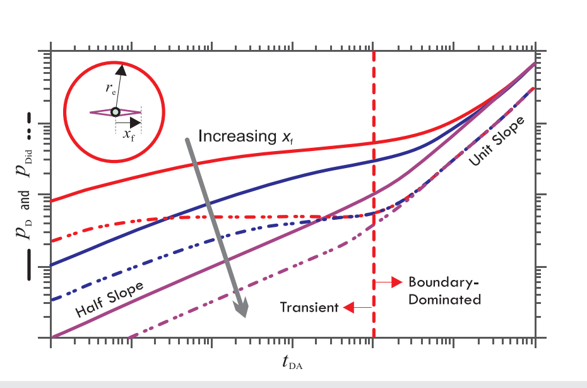


38-40: INFINITE CONDUCTIVITY FRACTURE

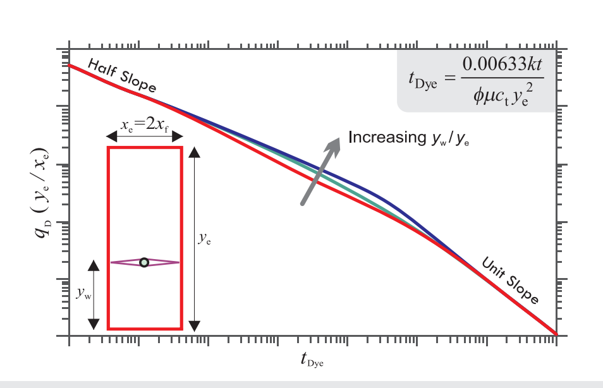
38. Blasingame: Rate and Integral-Derivative



39. NPI: Pressure and Integral-Derivative

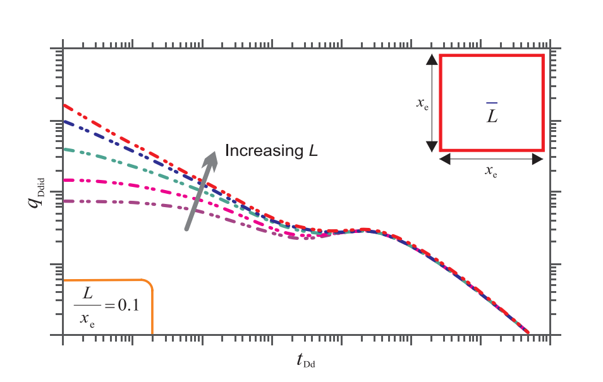


40. Wattenbarger: Rate

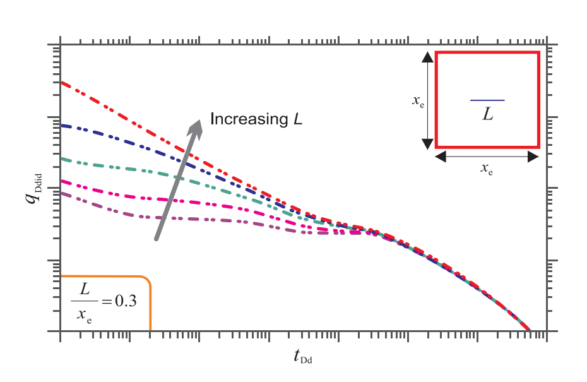


41-43: HORIZONTAL WELL TYPE CURVES

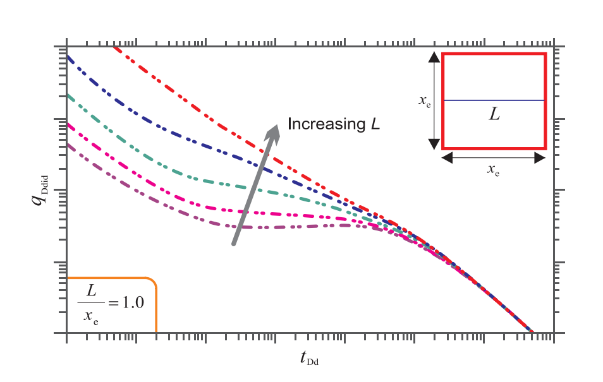
41. Blasingame: Integral-Derivative



42. Blasingame: Integral-Derivative

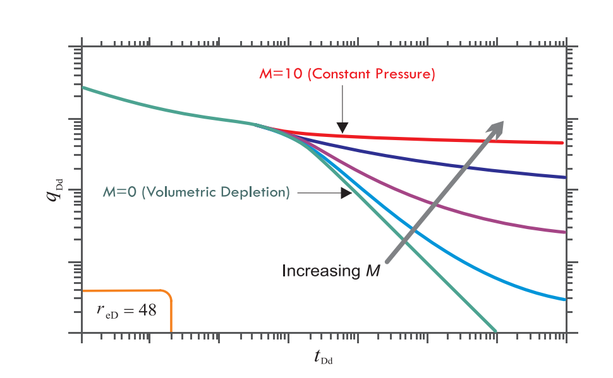


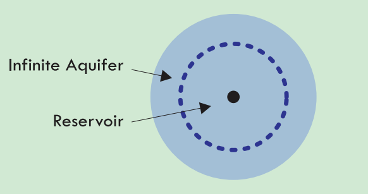
43. Blasingame: Integral-Derivative



44-45: WATER-DRIVE TYPE CURVES

44. Blasingame: Rate



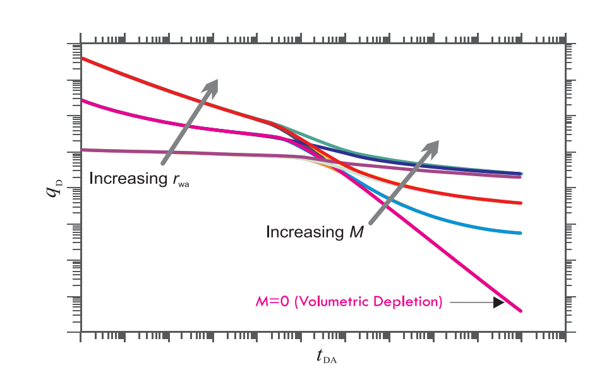


- Mobility ratio (M) represents the strength of the aquifer.

$$M=\frac{k_{aq}}{k_{res}}\frac{\mu_{res}}{\mu_{aq}}$$

- M = 0 is equivalent to Radial Type Curves (Figures 25-32).

45. Agarwal-Gardner: Rate



NOMENCLATURE

a	semi-major axis of ellipse
A	area
b	hyperbolic decline exponent or semi-minor axis of ellipse
b_{Dpss}	dimensionless parameter
b_{pss}	inverse of productivity index
B	formation volume factor
B_{gi}	initial gas formation volume factor
B_o	oil formation volume factor
B_{oi}	initial oil formation volume factor
c_g	gas compressibility
c_t	total compressibility
$\bar{c}_t$	total compressibility at average reservoir pressure
D	nominal decline rate
D_e	effective decline rate
D_i	initial nominal decline rate
F_{CD}	dimensionless fracture conductivity
G	original gas-in-place
G_p	gas cumulative production
G_{pa}	pseudo-cumulative production
h	net pay
k	permeability
k_{aq}	aquifer permeability
k_f	fracture permeability
k_h	horizontal permeability
k_{res}	reservoir permeability
k_v	vertical permeability
K	constant
L	horizontal well length
M	mobility ratio
N	original oil-in-place
N_p	oil cumulative production
p	pressure
$\bar{p}$	average reservoir pressure
p_0	reference pressure
p_D	dimensionless pressure
p_{Dd}	dimensionless pressure derivative
p_{Di}	dimensionless pressure integral
p_{Did}	dimensionless pressure integral-derivative
p_i	initial reservoir pressure
p_p	pseudo-pressure
$\bar{p}_p$	pseudo-pressure at average reservoir pressure
p_{pi}	initial pseudo-pressure
p_{pwf}	pseudo-pressure at well flowing pressure
p_{wf}	well flowing pressure
q	flow rate
q_D	dimensionless rate
q_{Dd}	dimensionless rate
q_{Ddi}	dimensionless rate integral
q_{Ddid}	dimensionless rate integral-derivative
q_i	initial flow rate
Q	cumulative production
Q_{Dd}	dimensionless cumulative production
r_e	exterior radius of reservoir
r_{eD}	dimensionless exterior radius of reservoir
r_w	wellbore radius
r_{wa}	apparent wellbore radius
s	skin
S_{gi}	initial gas saturation
S_{oi}	initial oil saturation
t	flow time
t_a	pseudo-time
t_c	material balance time
t_{ca}	material balance pseudo-time
t_D	dimensionless time
t_{DA}	dimensionless time
t_{Dd}	dimensionless time
t_{Dxf}	dimensionless time
t_{Dye}	dimensionless time
T	reservoir temperature
w	fracture width
x_e	reservoir length
x_f	fracture half length
y_e	reservoir width
y_w	well location in y-direction
Z	gas deviation factor
$\bar{Z}$	gas deviation factor at average reservoir pressure
Z_i	initial gas deviation factor
α	constant
ϕ	porosity
μ	viscosity
μ_{aq}	aquifer fluid viscosity
μ_g	gas viscosity
$\bar{\mu}_g$	gas viscosity at average reservoir pressure
μ_o	oil viscosity
μ_{res}	reservoir fluid viscosity

Oil field units; q_g (MMSCFD); t (days)

All analyses described can be performed using IHS Markit's Rate Transient Analysis software RTA.

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